

RIGHTSTART™ MATHEMATICS

by Joan A. Cotter, Ph.D.
with Kathleen Cotter Lawler
and Maren Stenseth Ehley

SIXTH GRADE LESSONS

Second Edition

Thank you to Kelsie Burza, Debbie Oberste, Kera Reich, and Rebecca Walsh for their work in the final preparation of this manual. A special thank you to students Alexis, Ben, Bonnie, Jaya, Josephine, Maia, Melody, Seth, Wesley, and Ysabella for their help with the second edition development.

Photo credits: Al Cotter, Andy Cotter, Constance Cotter, Joan A. Cotter, Maren Stenseth Ehley, Irene Genelín, Carrie Heidrich, Cherie Lou Kenner, Kathleen Cotter Lawler, and Kera Reich.

Copyright © 2018 by Activities for Learning, Inc.

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without written permission of Activities for Learning, Inc.

Printed in the United States of America

www.RightStartMath.com

For more information: info@RightStartMath.com
Supplies may be ordered from: www.RightStartMath.com

Activities for Learning, Inc.
321 Hill Street
Hazelton, ND 58544-0468
United States of America
888-775-6284 or 701-782-2000
701-782-2007 fax

ISBN 978-1-942943-33-4

March 2022

IMPORTANT NOTES TO THE STUDENT

Read Before You Start

Welcome to RightStart™ Mathematics Grade 6 Second Edition! This is an exciting time as you will be completing these lessons independently. We have put together some tips to help make this year as smooth as possible. Don't let the length of this section scare you. It's broken down into segments, so read it through and you will be ready to start!

LESSON MANUAL

This is the book you are reading right now. It contains your lessons. Take a moment to flip through the book. Notice the different sections for each lesson. Let's outline them for you.

- **Objectives:** These are the goals of the lesson. Pay close attention to the italicized words; these are terms you need to learn. You will find the explanation of those words in your lesson. Record the definitions in your Math Dictionary, found at the back of the Worksheets. Go find the dictionary now while you are thinking about it. We will talk more about this later.
- **Materials:** This list tells you what you will need for the lesson. Gather the items before you start. A 0.5 mm mechanical pencil is highly recommended. You will find more information on some of the materials used in these lessons as you keep reading.
- **Activities:** This is the lesson.
- **Extras:** This is, well, extras! It helps you with your lessons. It has tips and reminders placed next to the related paragraph in the Activities. Photos are also included showing you how these concepts are applied in real life.

This year, *you* are teaching yourself. You will be reading the lessons and worksheets and learning as you go. Your teacher is a facilitator. They will guide you, help you, and answer questions, but they will not be teaching you. Again, **YOU are teaching yourself**.

MATERIALS

Take a moment to check out the materials you will be using. Get familiar with their names and what they look like so you can quickly grab what you need for the lesson. There will be more detailed instructions in the lessons but for now, here's a list of the materials you will be using.

- **Drawing Board:** This is the surface you will tape your worksheets onto for your work. Using 3M Removable tape is highly recommended.
- **T-square:** The T-square is used to draw horizontal lines. It also provides alignment and support for the triangles so you can draw lines at the proper angles. Some T-squares may have measurements on them, but do not use the T-square as a ruler. It's a T-square!
- **30-60 Triangle and 45 Triangle:** These will be used to help you draw vertical lines and angled lines. The difference between the two triangles is important. The first lesson will identify the differences between the two triangles.

- **4-in-1 ruler:** This ruler has one side with measurements in centimeters and millimeters. The other side has measurements in tenths and sixteenths of an inch. A ruler with inches divided into tenths is unusual and you will find how useful it is!
- **Goniometer:** This tool is used to measure angles. It is best used when lying flat and is easiest to read if you lay it on white paper, such as your worksheet. The goniometer is numbered by tens. The lines between the numbers are by twos. If you are looking for 15° , move the upper arm of the goniometer until the indicator line in the bubble is halfway between the numbers 10 and 20, as well as halfway between the lines representing 4 and 6. Getting the exact measurement takes practice as it can be hard to see some of the numbers. Do your best.
- **mmArc™ Compass:** This tool is used to make circles and arcs. There are instructions in the lessons on how to use this compass.
- **Scientific Calculator:** The Casio Calculator fx-300MS calculator was carefully chosen for you. Lesson 15 will show you how to start using it. Don't be scared of all the fancy buttons. Throughout the year, the lessons will step you through the processes on how to use this calculator successfully.
- **Math Card Games book:** This book will tell you how to play the math card games to practice your arithmetic. Almost every lesson will assign a game to play. The games are important because they help you practice your math facts. The games are more fun than review sheets, so remember to play the games assigned.
- **Other materials:** There are other materials that you are most likely familiar with; tangrams, tape, and centimeter cubes. Geometry Panels are also included in your supplies, although they are not used in these lessons, they are used in RightStart™ Mathematics Grade 7.
- **Items listed in bold:** Every now and then, you will see items listed in bold. These things are not in your materials kit but things you generally have on hand.

ACTIVITIES AND EXTRAS

This section is your lesson and along with the worksheets will provide information for you to explore and learn.

1. **Read the lesson.** Unless it tells you otherwise, read through the lesson. Sometimes it will tell you to complete the worksheet, or part of the worksheet, before reading further. If it doesn't ask you to complete the worksheet before reading further, continue reading the entire lesson. Read it carefully!
2. **Look at the figures and pictures.** These accompanying figures enhance the lesson information. Sometimes a drawing can explain better than words. Pictures are also included and will show how the concepts you are learning are applied.
3. **Read the extras.** Here you will find hints and reminders to help with the lesson. Sometimes they are fun tidbits of information to enhance the lesson.

4. **Summarize.** Can you summarize what you have read? Do you have a good idea of what you will be doing? Great. But before you start on the worksheet you will need to do one more thing.... Don't skip this next step....

5. **Read through the lesson AGAIN!** This time look for terms you need to define. The terms are *italicized words* in the objectives and in the lesson.

It may help to underline or highlight the terms as well as its explanation in the lesson. Then immediately record the definition in your Math Dictionary, found at the back of your worksheets. Writing the definition in your own words helps you understand the terms better. There is also space to draw an example if needed. This may help you remember it better.

It is important to define these terms as you will be referring to these words throughout the lessons. Having the definitions in a central location will make it much easier when you need to check the meaning of something.

6. **Read through the lesson a THIRD time.** Now you are ready to begin the lesson. This will be your third time reading the same information.

Why are we telling you to read it so many times? No one learns mathematics by reading information only once. Honest. It doesn't matter how smart you are, reading more than once is needed. Often times, you will absorb more information and explanation as you read a second or third time. Sometimes you may skim over an important word the first time or miss a connection, but reading it the second or third time makes it clearer.

Review lessons. There are review lessons sprinkled throughout the year. These are not considered tests, therefore, you may refer to previous lessons if you need to look something up.

Assessments. Assessments are considered tests. You will not be allowed to look at lessons, worksheets, or your dictionary while completing an assessment.

Understanding. Each lesson needs to be understood before going to the next lesson. Don't go on if you don't understand what you are currently doing. The lessons often build on each other. If one lesson is shaky, the next one is going to be more challenging.

Questions. If you are having trouble understanding a lesson, don't just say, "I don't get it." That's not a question! How can anyone understand you and help you? You need to form a question to ask, then ask your facilitator. What exactly do you not understand? Be specific. Learning how to ask questions is an important skill to acquire toward becoming an independent learner.

WORKSHEETS

You have a separate binder that holds the worksheets. You will take the worksheet page out of the book, work on it, then return it to your binder. Each lesson will tell you which worksheet you will need. Some lessons will have one worksheet and other lessons will have more.

There are directions in Lesson 1 on how to tape and align the worksheet to the drawing board. You will be expected to do this with every worksheet, even though it's not written in the lessons.

Use a light hand when drawing on your worksheet. It's easier to erase things if you write lightly. You **will** be erasing things. This is very normal. A lighter line also makes for better accuracy.

Sometimes, when doing the worksheets, you will want to refer back to the lesson. You will be surprised at the hints or clues that make sense when you are actually doing the work.

Keep your completed worksheets in your binder. **Do not throw any worksheets away.** Some lessons will reference a prior worksheet. Sometimes you will want to refer back to them, especially when you get to the assessments.

CONTACT US

If you have read through the lesson at least three times, you have gone to your facilitator for help, and you are still confused, then call or email us directly. The email address, info@RightStartMath.com, and phone number, 888-RS5-MATH (888-775-6284), are on the back of this book. We will help you from 8:30 to 4:30 Central time, Monday through Thursday, and 8:30 to 12:30 on Fridays.

NEED TO KNOW

There is a Need to Know list located at the back of the worksheets, right before the Math Dictionary. These are important things you need to memorize.

The Need To Know list also tells you which lesson has the information. In the lesson, the Need to Know information will have a box around it in the Extras section. You may want to highlight this box with a specific color, different from what you are using for the definitions, as a reminder that this is important and you really NEED TO KNOW it. Plus, it's easier to find when looking for it.

MATH DICTIONARY

The Math Dictionary is located at the back of the worksheets, behind the Need to Know list. The terms are already written in the dictionary for you. It also tells you which lesson or lessons that term was introduced. A few of the terms will be found in more than one lesson because the meaning is expanded. It will be helpful to add new information to your initial definition.

Record the definition for the terms in your own words. Consider drawing a picture to better help you remember the definition.

GRADING

You will be grading your own work based on your persistence, understanding, and results. You are expected to do this after every lesson. In the Worksheet binder, look at the very first pages, titled Grade 6 Worksheet Results. Read the instructions.

Your facilitator has the Solutions manual. They may want to go over your work with you or they may allow you to look at the Solutions manual on your own.

Maybe you weren't totally sure about a concept but then it "clicked" after looking at the solutions. Maybe you want to redo your work. If needed, regrade yourself after looking at the solutions.

SOLUTIONS MANUAL

The Solutions are written to the facilitator. It will show the solutions to the worksheets as well as give additional explanations and insights. Your facilitator will decide how to use the Solutions book with you.

GAMES

Most lessons will have a math card game listed. These games are important because they help you practice your math facts. Make time to play them!

These games don't have to be played during the lesson. You can play the game right away or any time throughout the day. Some students will do their lesson then play the game at a later time. If you don't have someone to play the game with you, modify the game to be a solitaire game. Or you could have your right hand play against your left hand; in other words, play a game against yourself!

The important thing is that you are practicing your math facts. Playing a game for 10 to 15 minutes is the same as doing a worksheet. And it's a whole lot more fun than a drill of math facts!

IT'S TIME TO START.....

Now you're ready to begin. Turn the page and let's go!

Joan A. Cotter, Ph.D.
and Kathleen Cotter Lawler
and Maren Stenseth Ehley

IMPORTANT NOTES TO THE FACILITATOR

Welcome to RightStart™ Mathematics Grade 6 Second Edition! This is an innovative approach for teaching many middle school mathematics topics including fractions, perimeter, area, rounding, ratio, the metric system, pi, squares, square roots, and more. The student is also introduced to traditional geometric concepts: angles, rotations, reflections, symmetry, triangle congruence, Pythagorean Theorem, and so forth. In this program the student does not write out formal proofs, although they will learn to keep their processes organized and logical.

It is of prime importance to understand mathematics. Approaching mathematics through geometry gives the student an excellent way to understand and remember concepts. The hands-on activities create deeper learning. For example, to find the area of a triangle the student must first construct the altitude, measure it, and then perform the calculation.

The majority of the work is done with a drawing board, T-square, 30-60 triangle, 45 triangle, goniometer (a device for measuring angles), and mmArc™ Compass (a tool for drawing circles and arcs). It is interesting to note that CAD, Computer Aided Design, software is based on these drawing tools.

RightStart™ Mathematics Grade 6 Second Edition also incorporates other branches of mathematics as well as arithmetic and algebra. Some lessons have an art flavor, for example, constructing Gothic arches or circle designs. Other lessons have an engineering focus, like creating designs for car wheels. Even some history of mathematics is woven throughout the lessons.

This text is written with several goals for the student:

- a) to use mathematics previously learned,
- b) to learn to read math texts,
- c) to lay a good foundation for more advanced mathematics,
- d) to learn mathematical terms,
- e) to discover mathematics everywhere, and
- f) to enjoy mathematics.

INDEPENDENT STUDENT LEARNING

These lessons are written for the student to do independently. You are the facilitator, not the teacher. The lessons will teach the student and guide them to understanding. The worksheets are not a review, rather they are the discovery and application for the lesson at hand. When a student has a question, check with the Solutions and see if the notes provide information to guide the student to the solution.

We want our students to become independent learners. You may need to go through a transition time as you both adjust. Each student is different and some may need a little more guidance and assistance at first.

Encourage the students to read the lesson more than once to gain understanding. In the student's notes, they are told to read the lesson at least twice. Learning to read a math textbook is a necessary skill for success in advanced math classes.

One teacher had some students who were delayed readers. She let the students read the lesson alone and do the work. If they did well on their worksheets and could explain what they learned, they moved on. If not, she had them read the lesson and the directions on the worksheets aloud. Mistakes were often made because the students were skimming through the lesson and had missed a vital piece of information.

Sometimes they didn't pay attention to the detailed instructions. Because they were delayed readers, the students did not want to read the lesson more than once, so it took encouragement to help them understand it is normal to read the instructions multiple times. Learning to follow directions is a necessary skill for studying and everyday life.

If the student has a serious reading challenge, read the text aloud while they follow along then ask them to explain what they heard. Be sure each concept is understood.

For minor reading challenges, you might model aloud the process of reading a lesson, commenting on the figure, and summarizing the paragraph. Then ask the student to read the lesson aloud on their own. Sometimes the student needs support to overcome frustration, which is inherent in the learning process.

There are terms introduced and defined in the lessons for the student to write in their Math Dictionary. The dictionary is found at the back of the student's worksheets.

Encourage the student to use their words and drawings to define the new terms and concepts.

Math should be 95% understood and 5% memorized. Your student will find understanding concepts will help them recall the information. There is a Need to Know page for information or formulas that need to be memorized.

IMPORTANT NOTES TO THE STUDENT

In the student's Lesson book, there are pages right inside the cover to read. Make sure the student reads these before they begin. It would be beneficial for you to read it too. Some students may benefit from reading the pages aloud. This will ensure they have read it, as well as making the expectations clearly understood.

THE SOLUTIONS MANUAL

The solutions manual is written for you, the facilitator. It shows the solutions as well as gives additional insights into the lessons.

Read the Solutions manual for the lesson being worked on **before** the student does their lesson and worksheet. Read any special notes so you are prepared to answer questions the student may ask.

You may want to go over the work with your student or you may hand the manual to them and have them compare their work with the solutions. Some facilitators will let their student go over the solutions themselves then have them explain their work. Being able to point out mistakes is a great step in gaining better understanding and decreases the likelihood of repeating the same error in the future.

Keep in mind that some problems will have more than one solution. We will let you know in the Notes section, although we may not show you all the different solutions. Also keep in mind that when measuring, your student may not be as accurate as the solutions, therefore, their answers may differ. As long as their results are close to the given answers and the student understood the process, they don't need to redo their work. You may want to double check their arithmetic with a calculator. If there is a significant difference, help them go through the steps to see where they made their mistake. Sometimes it's as simple as measuring in inches rather than centimeters, forgetting part of their formula, or recording the answer incorrectly.

ENRICHMENT LESSONS

There are a few enrichment lessons sprinkled throughout the book. These lessons are optional, meaning it's up to you if you want the student to complete those lessons. The enrichment lessons expand on what the student has previously learned. It's best that they do these lessons, however, depending on the situation, it may not be necessary.

Enrichment lessons may include some new vocabulary. If you choose to not have your student complete these lessons, check if there are new terms introduced.

The last three lessons of RightStart™ Mathematics Grade 6 Second Edition are special Enrichment Lessons. These lessons take the student through steps to create an original design using a medium of their choice. The student will be using what they have learned throughout the year to create this design. It's advantageous for the student to complete these final lessons.

QUESTIONS, CORRECTIONS, AND SHARING

If you have any questions or concerns please have the **student** email us or call our office during business hours. Our contact information is on the back of this manual as well as the student's Lesson manual. We like feedback from you and the student.

If you or the student finds any errors in the books, please let us know. We do our best to proof this material, but unfortunately there still are errors that get overlooked. If you find a correction that needs to be made, email us at info@RightStartMath.com.

Occasionally, we encourage the student to send us their work so we can display it on our website. The student's parent or guardian must give permission to post the work. Also, indicate how to acknowledge the student: first name only, full name, region, or student's age. Send the work and information to info@RightStartMath.com and we will share their accomplishments.

Thank you for choosing to give your student a RightStart in Math!

Joan A. Cotter, Ph.D.
and Kathleen Cotter Lawler
and Maren Stenseth Ehley

SIXTH GRADE TABLE OF CONTENTS

Lesson 1:	Getting Started
Lesson 2:	Drawing Diagonals
Lesson 3:	Drawing Stars
Lesson 4:	Equilateral Triangles into Halves
Lesson 5:	Equilateral Triangles into Sixths & Thirds
Lesson 6:	Equilateral Triangles into Fourths & Eighths
Lesson 7:	Equilateral Triangles into Ninths
Lesson 8:	Equilateral Triangles into Twelfths and More
Lesson 9:	Hexagrams and Seal of Solomon
Lesson 10:	Review and Games 1
Lesson 11:	Measuring Perimeter in Centimeters
Lesson 12:	Enrichment Measuring Perimeter in Inches
Lesson 13:	Drawing Parallelograms in Centimeters
Lesson 14:	Enrichment Drawing Parallelograms in Inches
Lesson 15:	Scientific Calculator and Perimeter Formula
Lesson 16:	Converting Inches to Centimeters
Lesson 17:	Drawing Rectangles
Lesson 18:	Drawing Rhombuses
Lesson 19:	Drawing Squares
Lesson 20:	Classifying Quadrilaterals
Lesson 21:	Review and Games 2
Lesson 22:	The Fraction Chart
Lesson 23:	Patterns in Fractions
Lesson 24:	Measuring With Sixteenths
Lesson 25:	A Fraction of Geometry Figures
Lesson 26:	Making the Whole

SIXTH GRADE TABLE OF CONTENTS

Lesson 27:	Review and Games 3
Lesson 28:	Square Centimeters
Lesson 29:	Square Inches
Lesson 30:	Area of a Rectangle
Lesson 31:	Comparing Areas of Rectangles
Lesson 32:	Enrichment Product of a Number + Two More
Lesson 33:	Enrichment Area of Consecutive Squares
Lesson 34:	Area of a Parallelogram
Lesson 35:	Rounding
Lesson 36:	Comparing Calculated Areas of Parallelograms
Lesson 37:	Area of a Triangle
Lesson 38:	Comparing Calculated Areas of Triangles
Lesson 39:	Review and Games 4
Lesson 40:	Review for Assessment 1
Lesson 41:	Assessment 1
Lesson 42:	Name that Figure
Lesson 43:	Area of Trapezoids
Lesson 44:	Area of Hexagons
Lesson 45:	Area of Stars
Lesson 46:	Area of Octagons
Lesson 47:	Ratios and Nested Squares
Lesson 48:	Finding the Areas of More Triangles
Lesson 49:	Ratios of Areas
Lesson 50:	Measuring Angles
Lesson 51:	Supplementary and Vertical Angles
Lesson 52:	Angles in a Polygon

SIXTH GRADE TABLE OF CONTENTS

Lesson 53:	Review and Games 5
Lesson 54:	Classifying Triangles by Sides and Angles
Lesson 55:	External Angles of a Triangle
Lesson 56:	Angles Formed With Parallel Lines
Lesson 57:	Triangles With Congruent Sides (SSS)
Lesson 58:	Other Congruent Triangles (SAS, ASA)
Lesson 59:	Side and Angle Relationships in Triangles
Lesson 60:	Medians in Triangles
Lesson 61:	More About Medians in Triangles
Lesson 62:	Relationships of Medians in Triangles
Lesson 63:	Connecting Midpoints in a Triangle
Lesson 64:	Rectangles Inscribed in a Triangle
Lesson 65:	Connecting Midpoints in a Quadrilateral
Lesson 66:	Review and Games 6
Lesson 67:	Introducing the Pythagorean Theorem
Lesson 68:	Squares on Right Triangles
Lesson 69:	Proofs of the Pythagorean Theorem
Lesson 70:	Finding Square Roots
Lesson 71:	More Right Angle Problems
Lesson 72:	The Square Root Spiral
Lesson 73:	Square Root Spiral Pattern
Lesson 74:	Review and Games 7
Lesson 75:	Review for Assessment 2
Lesson 76:	Assessment 2
Lesson 77:	Circle Basics
Lesson 78:	Inscribed Polygons

SIXTH GRADE TABLE OF CONTENTS

Lesson 79:	Tangents to Circles
Lesson 80:	Circumscribed Polygons
Lesson 81:	Ratio of Circumference to Diameter
Lesson 82:	Pi, a Special Number
Lesson 83:	Circle Designs
Lesson 84:	Rounding Edges With Tangents
Lesson 85:	Making a “No U-Turn” Sign
Lesson 86:	Tangent Circles
Lesson 87:	Review and Games 8
Lesson 88:	Bisecting Angles
Lesson 89:	Inscribed Circles
Lesson 90:	Perpendicular Bisectors
Lesson 91:	The Amazing Nine-Point Circle
Lesson 92:	Drawing Arcs and Tangents
Lesson 93:	More Drawing Arcs
Lesson 94:	Angles ‘n Arcs
Lesson 95:	Arc Length
Lesson 96:	Review and Games 9
Lesson 97:	Area of a Circle
Lesson 98:	Finding the Area of a Circle
Lesson 99:	Babylonian Circles
Lesson 100:	Finding More Areas
Lesson 101:	Doubling Perimeter, Circumference, & Area
Lesson 102:	Pizza Problems
Lesson 103:	Review and Games 10
Lesson 104:	Revisiting Tangrams

SIXTH GRADE TABLE OF CONTENTS

Lesson 105:	Aligning Objects
Lesson 106:	Reflecting
Lesson 107:	Rotating
Lesson 108:	Making Wheel Designs
Lesson 109:	Identifying Reflections and Rotations
Lesson 110:	Review and Games 11
Lesson 111:	Translations
Lesson 112:	Transformations
Lesson 113:	More Transformations
Lesson 114:	Double Reflections
Lesson 115:	Finding the Line of Reflection
Lesson 116:	Review and Games 12
Lesson 117:	Finding the Center of Rotation
Lesson 118:	More Centers of Rotation
Lesson 119:	More Double Reflections
Lesson 120:	Angles of Incidence and Reflection
Lesson 121:	Lines of Symmetry
Lesson 122:	Rotational Symmetry
Lesson 123:	Symmetry Connections
Lesson 124:	Frieze Patterns
Lesson 125:	Review and Games 13
Lesson 126:	Review for Assessment 3
Lesson 127:	Assessment 3
Lesson 128:	Enrichment Year End Original Designs
Lesson 129:	Enrichment Design Plans
Lesson 130:	Enrichment Design Creation

LESSON 8: EQUILATERAL TRIANGLE INTO TWELFTHS AND MORE

OBJECTIVES:

- 1. To divide an equilateral triangle into twelfths
- 2. To divide an equilateral triangle into a number greater than 12

MATERIALS:

- 1. Worksheets 8-1 and 8-2, Equilateral Triangle into Twelfths and More
- 2. Drawing board, T-square, and 30-60 triangle
- 3. **Colored pencils, optional**
- 4. *Math Card Games* book, F10

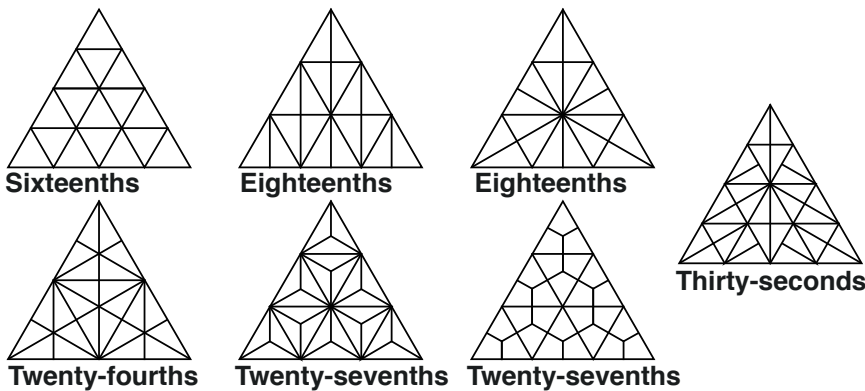
ACTIVITIES:

Dividing a triangle into twelfths. How would you divide an equilateral triangle into twelfths, that is, into twelve congruent parts? Think about it for a while before reading further.

Would it work to divide the triangle into thirds and divide each third into fourths? One student suggested dividing the triangle into tenths and then dividing each tenth in half. Let’s hope he was joking!

As you thought about it, you probably realize you first divide the triangle into fourths and then each fourth into thirds. The fourths create equilateral triangles, which allows you to continue to divide.

Dividing a triangle by higher numbers. How would you divide a triangle into sixteenths? Could you divide it into sixteenths another way? Answers are at the bottom of the page. Two kindergarten girls divided an equilateral triangle into 256 equal parts. After hearing about the girls, a teacher divided his triangle into 432 equal parts. Some divisions are shown below.



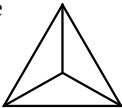
Worksheet 8-1. For this worksheet, you will divide the equilateral triangle into congruent twelfths. Work carefully. For Problem 2, figure out how you would divide equilateral triangles into various congruent pieces.

Worksheet 8-2. After drawing the equilateral triangle, divide it into congruent triangles. Either copy one of the designs above, or better yet, design your own. You might like to color your design.

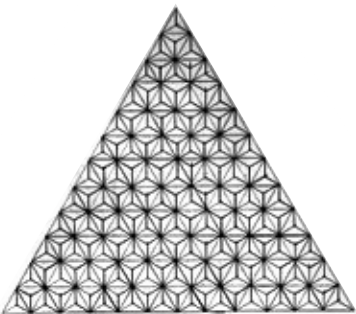
Today’s game. Play the Fraction of Twelve game, found in the *Math Card Games* book, F10.

EXTRAS:

If first dividing a triangle into thirds, it is impossible to divide the thirds into congruent fourths.



When dividing a triangle into fourths, each of these equilateral triangles can be divided into congruent thirds, so all twelfths are congruent.

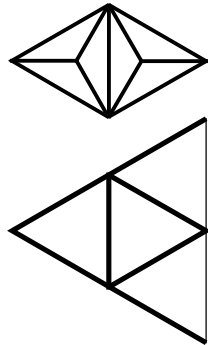


Triangle into 432nds by Joseph Hermodson-Olsen, age 14.
How could he have done it?
The answer is at the bottom of the page.

When dividing a triangle, create equilateral triangles, which then allows further divisions.

Answers: 16ths – fourths then fourths.
No other possibilities.
432nds – ninths, fourths, and thirds.

1. Draw an equilateral triangle. Divide it into fourths. Then divide each fourth into thirds, as shown.



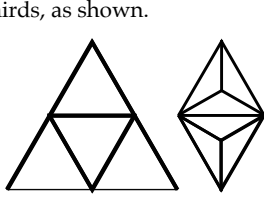
Name: _____

Date: _____

2. Fill in the chart. Reread the lesson for help.

Number of Pieces	First Division	Second Division	Third Division
8	4	2	
12	4		
16			
18			
24			
27			
48			
	4	4	4
	9	9	

1. Draw an equilateral triangle. Divide it into fourths. Then divide each fourth into thirds, as shown.



2. Fill in the chart. Reread the lesson for help.

Number of Pieces	First Division	Second Division	Third Division
8	4	2	
12	4	3	
16	4	4	
18	9	2	
24	4	3 [6]	2 [blank]
27	9	3	
48	4	4	3
64	4	4	4
81	9	9	

NOTES: The order of division is very important. Because congruent pieces are needed, the divisions must be equilateral triangles if additional divisions are needed. Let's look at some triangle divisions and see what options exist.



Division into halves: Once it's divided, it cannot be divided again and still have congruent parts.



Division into thirds: Regardless of which division into thirds is made, only one more division into halves is possible. If any other division is made, the parts will not be congruent.



Division into fourths: This creates equilateral triangles. This allows for many options for additional divisions.



Division into sixths: This cannot be divided again and create congruent parts.



Division into eighths: This cannot be divided again and create congruent parts.



Division into ninths: Again, we have equilateral triangles. Many options are available from this point.

Notice that if a second or third division of the triangle is necessary, the preceding division must create equilateral triangles by dividing into fourths or ninths. The exception is dividing by thirds, then halves as the last division.

LESSON 45: AREA OF STARS

OBJECTIVES:

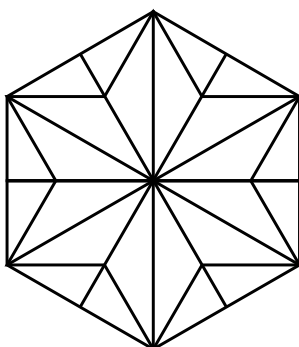
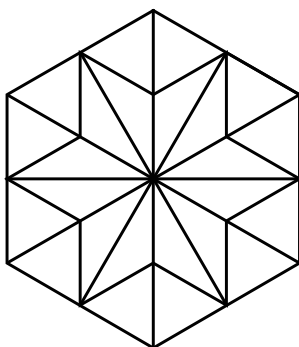
1. To calculate the areas of stars inside a hexagon

MATERIALS:

1. Worksheets 45-1 and 45-2, Area of Stars
2. Drawing board, T-square, and 30-60 triangle
3. 4-in-1 ruler
4. Casio Calculator fx-300MS
5. *Math Card Games* book, D14

ACTIVITIES:

Area of a star. In one of the very beginning lessons, you drew stars in hexagons. On the two worksheets for today, you will be drawing the same stars again. The boldfaced lines in the little figures tell you what to draw. Be sure to use your T-square and 30-60 triangle to draw all the lines. The completed stars will look like the figures below.



Worksheets 45-1 and 45-2. Draw the stars and estimate the area in cm^2 . Divide the stars into polygons whose areas you can calculate then find the area of each star.

Today's game. Play the Mixed Multivide game, found in the *Math Card Games* book, D14.

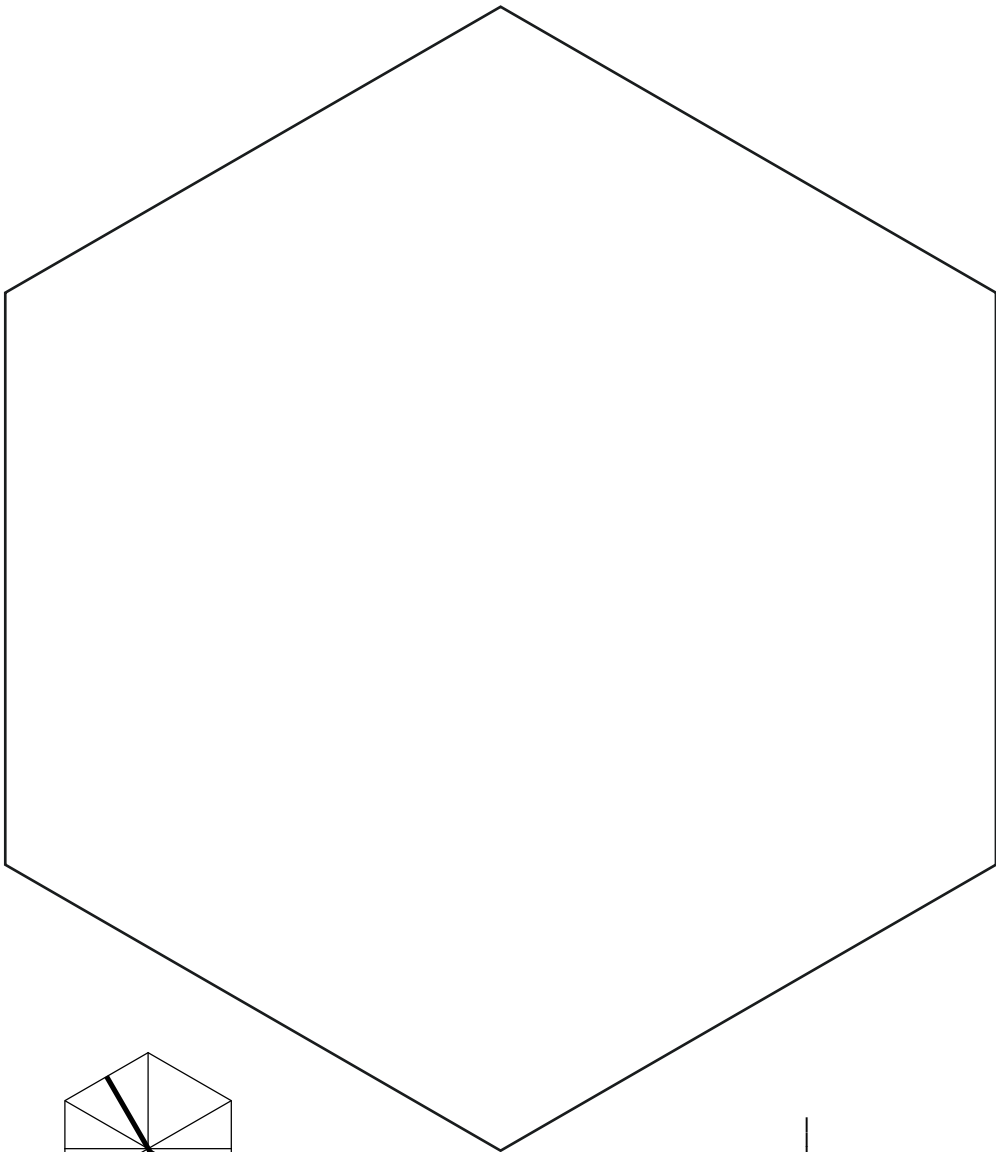
EXTRAS:



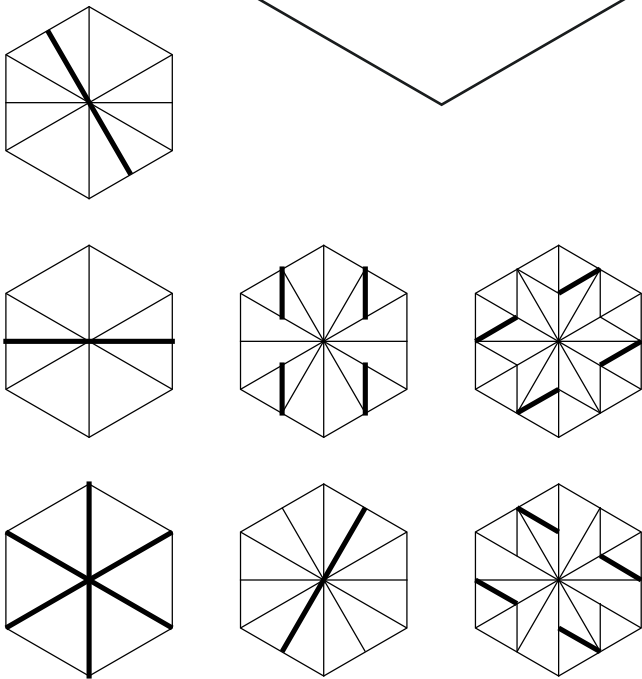
Star pattern on a building.

Name: _____

Date: _____



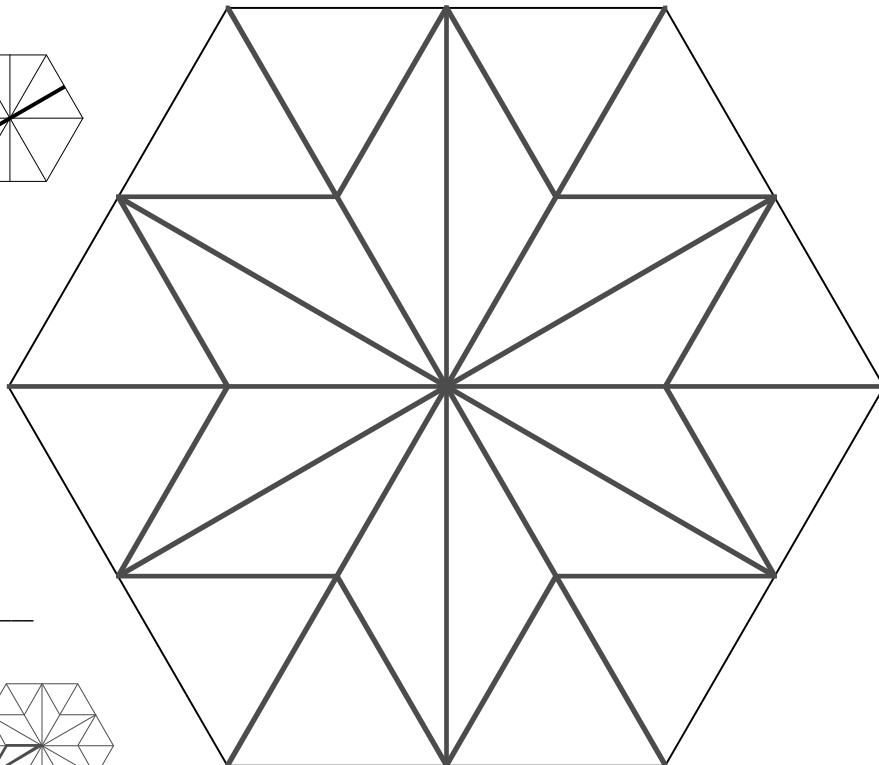
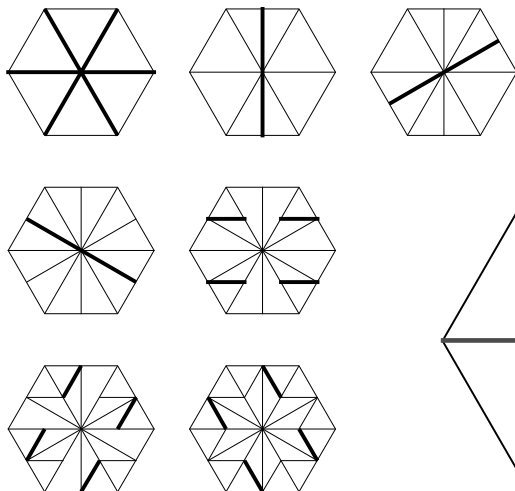
1. Draw a star by using your drawing tools and following the instructional figures below.



2. Guess the area of the star in cm^2 . _____

3. Find the area of the star in cm^2 .

1. Draw a star by using your drawing tools and following the instructional figures below.



2. Guess the area of the star in cm^2 . _____

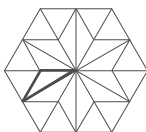
3. Find the area of the star in cm^2 .

$$A(\text{star}) = 12 \times A(\text{tri})$$

$$= 12 \times \frac{1}{2}wh$$

$$= 12 \times \frac{1}{2} \times 6.6 \times 1.9$$

$$= 75.2 \text{ cm}^2$$



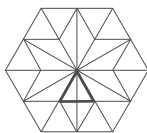
NOTES: Student's measurements may vary slightly, which will affect the calculated area. Some students are concerned with the precision of the answer. Without rounding, the answer is 75.24 cm^2 . Rounding to tenths gives 75.2 cm^2 , as provided here, and rounding to the nearest cm gives 75 cm^2 . Frequently, in real life situations, the specified precision of the answer is not provided. It is up to the individual to determine the needed accuracy.

There are a number of other ways to find the area of the star. Here are some options.

$$\begin{aligned} 1. A(\text{star}) &= A(\text{hex}) - 12 A(\text{eq tri}) \\ &= 6 A(\text{lg tri}) - 12 A(\text{eq tri}) \\ &= 6 \times \frac{1}{2}wh - 12 \times \frac{1}{2}wh \\ &= 6 \times \frac{1}{2} \times 7.6 \times 6.6 - 12 \times \frac{1}{2} \times 3.8 \times 3.3 \\ &= 150.48 - 75.24 = 75.2 \text{ cm}^2 \end{aligned}$$



$$\begin{aligned} 2. A(\text{star}) &= 12 A(\text{eq tri}) \\ &= 12 \times \frac{1}{2}wh \\ &= 12 \times \frac{1}{2} \times 3.8 \times 3.3 \\ &= 75.2 \text{ cm}^2 \end{aligned}$$



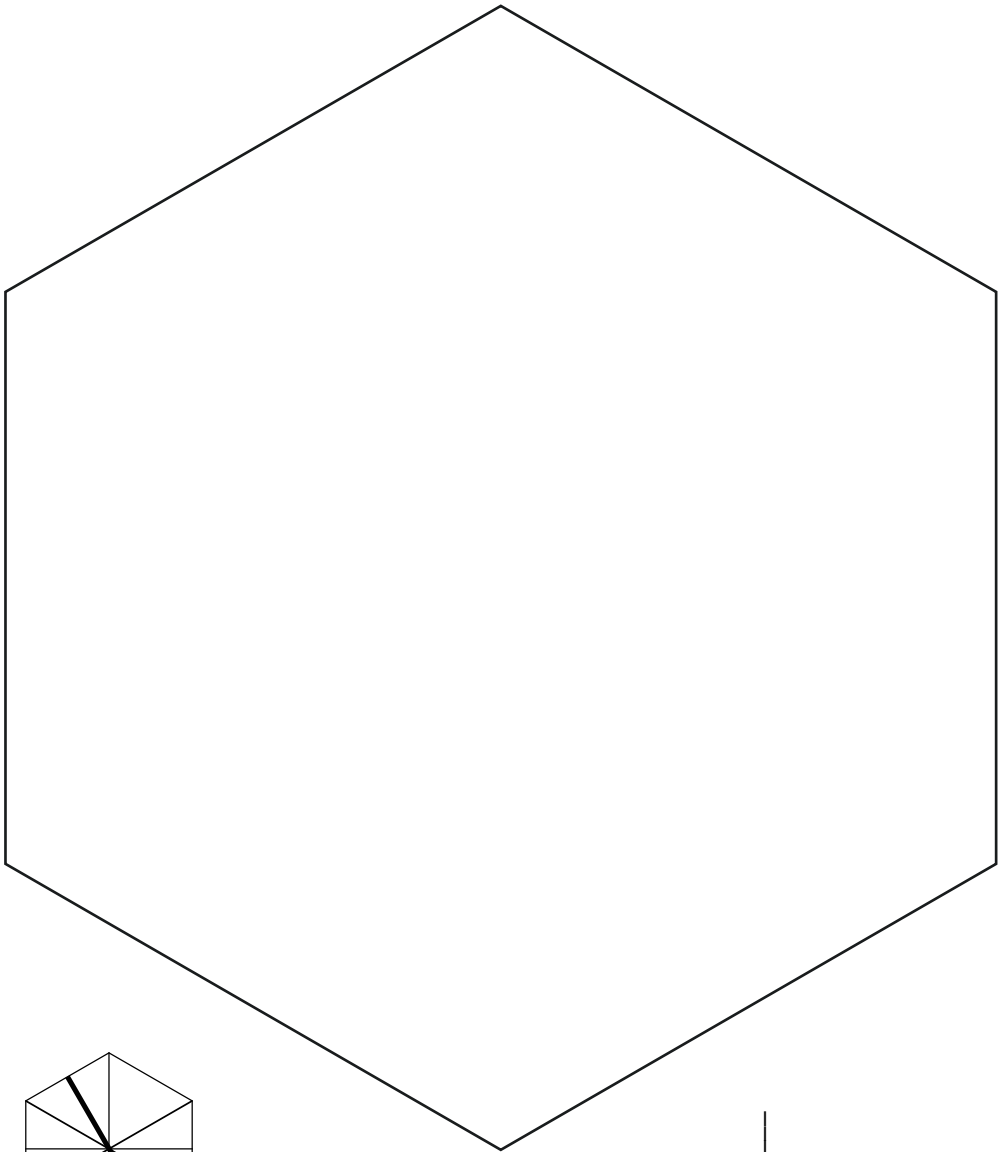
$$\begin{aligned} 3. A(\text{star}) &= 6 A(\text{parallelogram}) \\ &= 6wh \\ &= 6 \times 3.8 \times 3.3 \\ &= 75.2 \text{ cm}^2 \end{aligned}$$



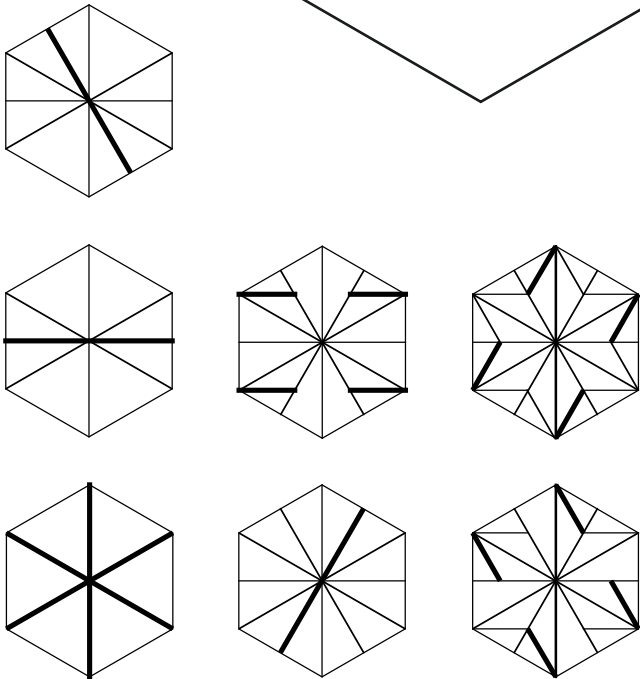
Some students may see that the star is half the area of hexagon, 12 equilateral triangles in the star and 24 total in the hexagon. Once the area of hexagon is calculated, 150.48 cm^2 , dividing in half gives the area of the star.

Name: _____

Date: _____



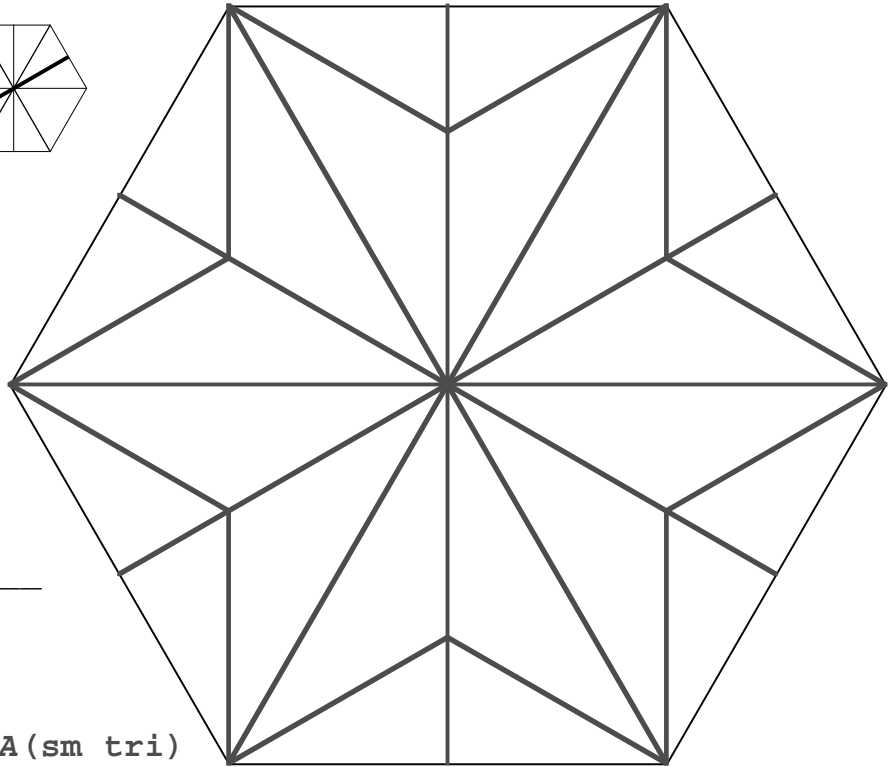
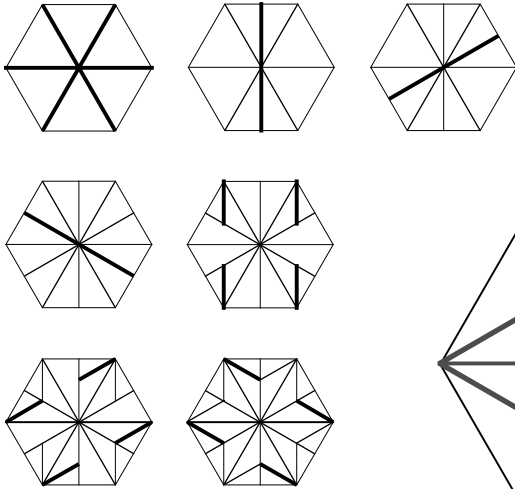
4. Draw a star by using your drawing tools and following the instructional figures below.



5. Guess the area of the star in cm^2 . _____

6. Find the area of this star in cm^2 .

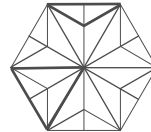
4. Draw a star by using your drawing tools and following the instructional figures below.



5. Guess the area of the star in cm^2 . _____

6. Find the area of this star in cm^2 .

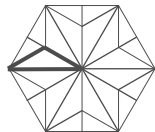
$$\begin{aligned}
 A(\text{star}) &= A(\text{hex}) - 6 \times A(\text{sm tri}) \\
 &= 6 \times A(\text{lg tri}) - 6 \times A(\text{sm tri}) \\
 &= 6 \times \frac{1}{2}wh - 6 \times \frac{1}{2}wh \\
 &= 6 \times \frac{1}{2} \times 7.6 \times 6.6 - 6 \times \frac{1}{2} \times 7.6 \times 2.2 \\
 &= 150.48 - 50.16 = 100.3 \text{ cm}^2
 \end{aligned}$$



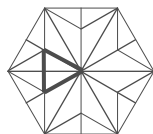
NOTES: Like Worksheet 44-1, there are a number of other ways to find the area of the star. Considering the solution given above, some students might quickly realize the area of the two hexagons, this one and the one from Worksheet 44-1, are the same. Therefore, they will have the same area of 150.48 cm^2 . Others may not recognize that. For those students that did not recognize the congruency, they got to practice their arithmetic. For those who did, good thinking!

Here are some more options to finding the area of this star.

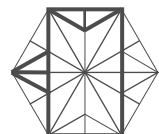
$$\begin{aligned}
 1. A(\text{star}) &= 12 A(\text{med tri}) \\
 &= 12 \times \frac{1}{2}wh \\
 &= 12 \times \frac{1}{2} \times 7.6 \times 2.2 \\
 &= 100.3 \text{ cm}^2
 \end{aligned}$$



$$\begin{aligned}
 2. A(\text{star}) &= 12 A(\text{eq tri}) \\
 &= 12 \times \frac{1}{2}wh \\
 &= 12 \times \frac{1}{2} \times 4.4 \times 3.8 \\
 &= 100.3 \text{ cm}^2
 \end{aligned}$$



One student, Clayton, simply calculated the area of the rectangle. He saw that four triangles included at the top and the bottom of the rectangle are congruent to the four needed triangles that are excluded from the rectangle.



DICTIONARY TERMS: none

LESSON 47: RATIOS AND NESTED SQUARES

OBJECTIVES:

1. To learn the term *ratio*
2. To learn to write ratios
3. To construct a series of nested squares

MATERIALS:

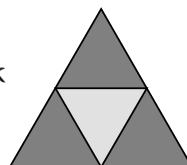
1. Math Dictionary
2. Worksheet 47, Ratios and Nested Squares
3. Drawing board, T-square, and 45 triangle
4. **Scissors**
5. *Math Card Games* book, D14

ACTIVITIES:

Ratios. Whether you've heard of ratios or not, they're not very hard. A *ratio* (RAY-shee-o) is two numbers telling how many times greater or smaller one object is compared to another. There are three types of ratios and three ways to write a ratio.

An example is the best way to understand ratios. In the figure at the right, the ratio of the area of the dark triangles to the area of the large triangle is 3 to 4.

This is a **part to whole ratio**.



The ratio of the area of the large triangle to the small light triangle is 4:1, read as 4 to 1. This is a **whole/part ratio**, the second type of ratio.

The ratio of the small light triangle to the dark triangles is 1 to 3. It can also be read as $\frac{1}{3}$. That is a **part/part ratio**, the third type of ratio.

All this is summarized in the chart below.

Type	Triangles (area)	Ways to write ratios		
Part to whole	Dark to large	3 to 4	3:4	$\frac{3}{4}$
Whole to part	Large to light	4 to 1	4:1	$\frac{4}{1}$
Part to part	Light to dark	1 to 3	1:3	$\frac{1}{3}$

Types of ratios and ways to write them, using the figure above as the example.

Worksheet 47. Start the worksheet halfway down the page, instead of at the top. This strange beginning allows you to cut part of the page while leaving enough paper to draw the day's masterpiece. Draw the square. Refer to Worksheet 19 if you need a refresher.

To draw the smaller square, square B, first find the larger square's center. This is shown on the left at the top of the next page. Then you can find the midpoint of the top line. See the figures on the next page.

EXTRAS:

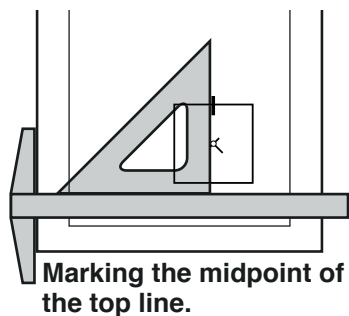
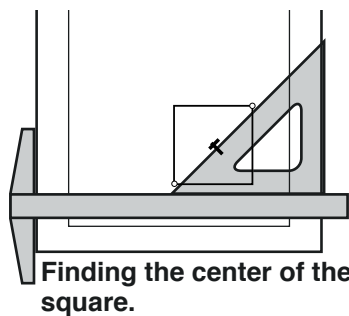
There are three dark triangles. The total number of triangles is four. So the ratio of dark triangles to the large triangle is 3 to 4.

Some people confuse the quantity of triangles with the area of the triangles. Notice you are looking at the **area** of the large triangle to the **area** of the little light colored triangle, 4:1.

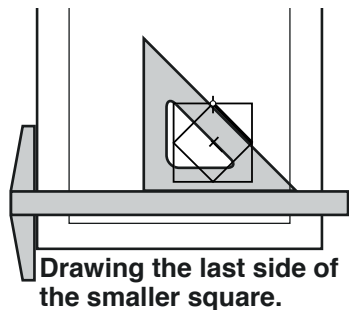
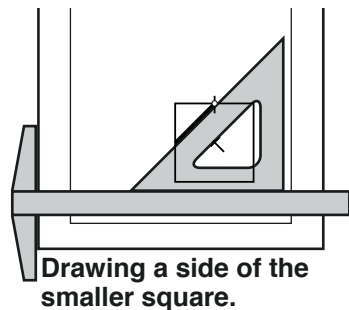
LESSON CONTINUES ON
THE NEXT PAGE.

ACTIVITIES:

EXTRAS:

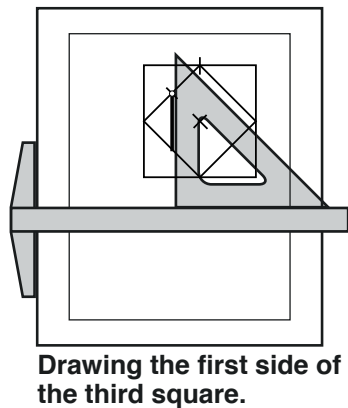
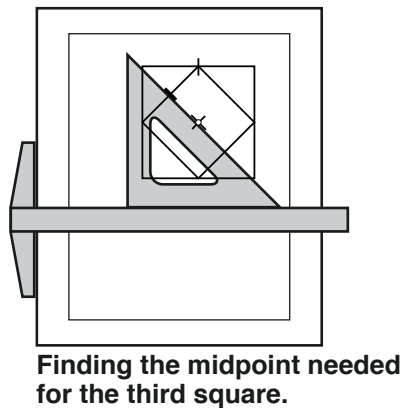


The square is completed by following these steps.



Cut out the large square and fold on the lines of the small square. What do you notice? What is the ratio of the area of the smaller square to the larger square? Answers are at the bottom of the page. Now cut out the small square, saving the triangles. Arrange all four triangles into one square. How does it compare to the small square? Finally, take two of the triangles and form a square. What is the ratio of the smallest square to the largest square? The answers are at the bottom of the page.

Nested squares. To draw the third square in Question 4, square C, find the midpoint of the side of square B. It lies on square A's diagonal. See the left figure below. Draw the first side as shown below in the right figure. Finish square C and draw as many additional nested squares as you can.



Today's game. Play the Mixed Multivide game, found in the *Math Card Games* book, D14.

Puzzle for you: Use the five pieces – the square and the four triangles – to make a) one square, b) two squares, or c) three squares.

Watch for cascading errors. This happens when one tiny error is introduced then another error based on the first error becomes a bigger error and the situation cascades out of control.

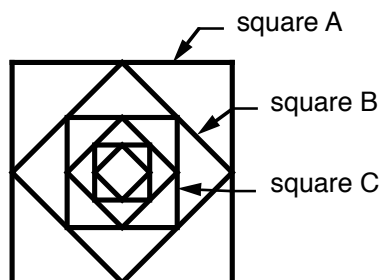
Answers: triangles cover small square, ratio 1 to 2 squares are congruent, ratio 1 to 4

Name: _____

Date: _____

START HALFWAY DOWN THE PAGE.

4. Using the line, draw the nested square figure shown below. Continue the pattern as far as you can.



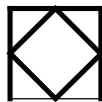
5. What is the ratio of area of square B to square C?

6. What is the ratio of area of square A to square C?

START HERE.

1. Using the line at the bottom right, construct a square. Next connect the midpoints of the sides as shown. Cut out the large square and follow the activities in the lesson.

When you are finished, save the pieces. You might want to attach them to this page with removable tape.

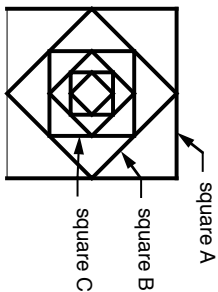


2. What is the ratio of the area of the outer square to the inner square? _____

3. What is the ratio of the area of one triangle to the inner square? _____

START HALFWAY DOWN THE PAGE.

4. Using the line, draw the nested square figure shown below. Continue the pattern as far as you can.



5. What is the ratio of area of square B to square C?

2 : 1

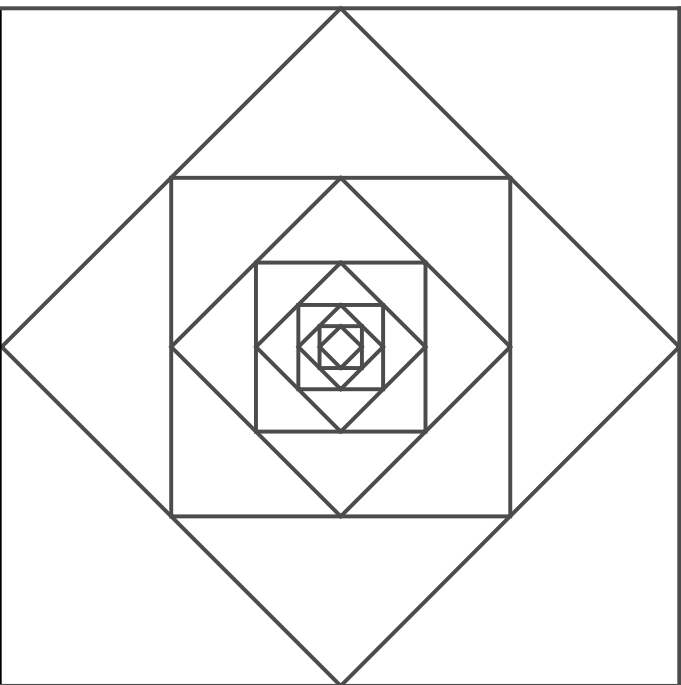
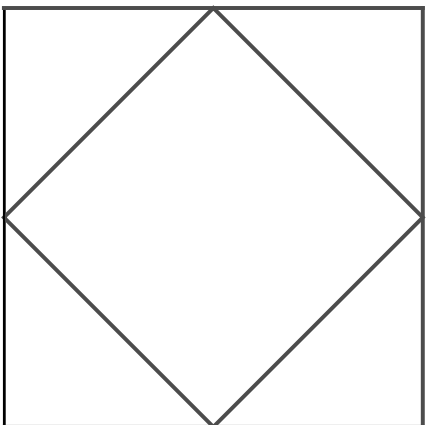
6. What is the ratio of area of square A to square C?

4 : 1

START HERE.

1. Using the line at the bottom right, construct a square. Next connect the midpoints of the sides as shown. Cut out the large square and follow the activities in the lesson.

When you are finished, save the pieces. You might want to attach them to this page with removable tape.



2. What is the ratio of the area of the outer square to the inner square? **2 : 1**
3. What is the ratio of the area of one triangle to the inner square? **1 : 4**

NOTES: Make sure tick marks are displayed with the student's work. Extra lines that extend beyond the square should be erased.

Here is Josephine Wilson's work. See how the tick marks are visible, yet no extra lines are included?

The second drawing here is an example of cascading errors. Notice how one error compounds the next error.

Questions 2 and 3 can be answered as 2:1, 2 to 1, or $\frac{2}{1}$ and 1:4, 1 to 4, or $\frac{1}{4}$.

Make sure the student is paying attention to the order that the items are listed when discussing ratios. Considering

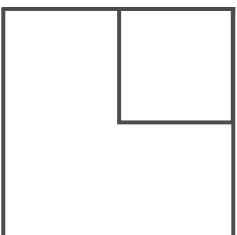
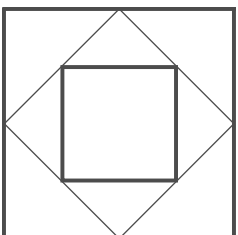
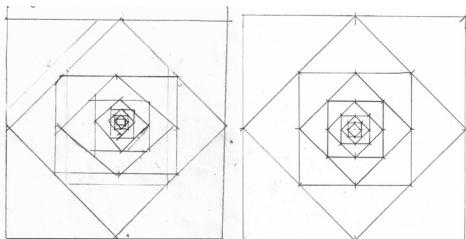
Question 2; if it was asking for the ratio of the inner square to the outer square, the answer would be 1:2. However, it asks for the ratio of outer square to inner square which is 2:1. The order is critical.

Question 5, the ratio of square B to square C, could also be answered as 8:4, which is the same as 2:1.

Question 6 asks the ratio of square A to square C. There are two ways to approach this. The first follows the lesson. Square B is half the size of square A and square C is half the size of square B. Half of a half is one-fourth. Therefore, the ratio of square C to square A is 1:4 and the ratio of square A to square C is 4:1.

Another way to approach this is visually. Imagine moving square C up to the corner of square A. Now it's easier to see that the ratio of square A to square C is 4:1 because four square Cs will fit inside square A. "Eyeballing" to get the answer isn't always accurate, but sometimes it helps to gain perspective.

DICTIONARY TERMS: ratio



LESSON 82: PI, A SPECIAL NUMBER

OBJECTIVES:

1. To learn the term, *pi*, and its symbol, π
2. To learn some simple approximations for π : 3.14, $3\frac{1}{7}$, and $\frac{22}{7}$
3. To learn the symbol \approx
4. To calculate circumferences, diameters, and radii using π

MATERIALS:

1. Math Dictionary
2. Need to Know Information
3. Worksheets 82-1 and 82-2, Pi, a Special Number
4. 4-in-1 ruler
5. Casio Calculator fx-300MS
6. *Math Card Games* book, D6

ACTIVITIES:

Pi. In the last lesson, you found how many times the diameter fits around a circle. The number, about 3.14, is the same for all circles. It is also the ratio of the circumference to the diameter.

Since the 1600s this ratio, circumference to diameter, has been represented by the Greek letter π (for perimeter), written π . We pronounce it as “pie” and spell it *pi*.

Making pi. To write the π symbol, see the figure at right. First draw an almost vertical line. A short distance away, draw another line with a hook at the bottom. Lastly, draw the top with a hook on the left.

More pi. Pi has interested mathematicians for over 2500 years. Several books have been written about this number, including one for children, *Sir Cumference and the Dragon of Pi: A Math Adventure* by Cindy Neuschwander. In a Toronto subway there is a tiled wall, designed by Arlene Stamp, based on pi. There is even pi music! For fun, check out Lars Erickson’s *Pi Symphony*.

Pi is not an exact number. It is close to 3.14. If we write $\pi \approx 3.1416$, it is more accurate but still not exact. The symbol \approx means approximately, or about.

If we write it as $\pi \approx 3.141592653$, it is closer but still not exact. In 2002, pi was calculated to over 1 trillion digits. Even with this many digits, it’s still not exact!

Values of pi. Working with more than a few digits of pi is generally not necessary. For most problems, use one of these approximations: $\pi \approx 3.14$, or $\frac{22}{7}$, or $3\frac{1}{7}$

Formulas for circumference. The following formula for finding circumference should make sense to you, where C is circumference and D is diameter:

$$C = \pi D$$

Since a radius is half a diameter, the formula using a radius is:

$$C = 2\pi r$$

Worksheets 82-1 and 82-2. Now apply these concepts on the worksheets.

Today’s game. Play the Equal Quotients game, found in the *Math Card Games* book, D6.

EXTRAS:

π

Kera’s funny for the day: An opinion without 3.14 is just an onion.

You can listen to the *Pi Symphony* at RightStartMath.com/geometry.

People have memorized pi to hundreds of places. According to the Seattle Times (2-26-1995), Hiroyuki Goto, then age 21, memorized over 42,000 digits of pi. It took him over nine hours to recite it.

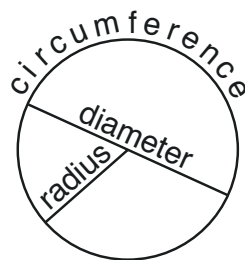
NEED TO KNOW:

$$\pi \approx 3.14, \text{ or } \frac{22}{7}, \text{ or } 3\frac{1}{7}$$

NEED TO KNOW:

$$C = \pi D \text{ or } C = 2\pi r$$

Remember from Lesson 30 that two letters or symbols written together without an operator means to multiply.

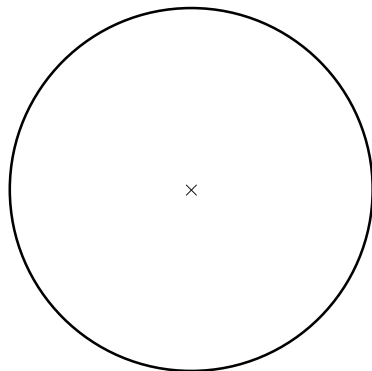


Name: _____

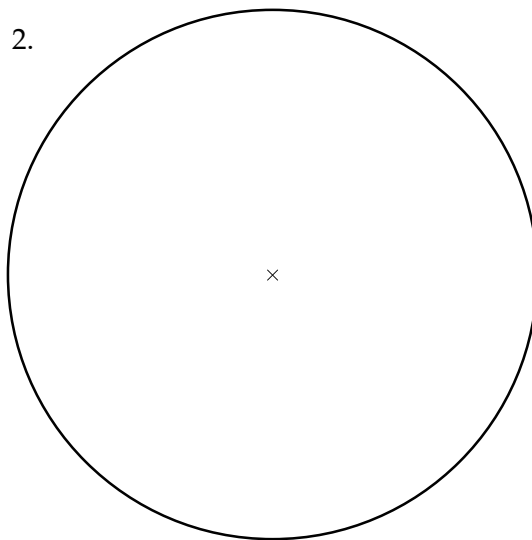
Date: _____

Draw and measure the diameters of the following circles and calculate the circumferences in cm. Use the fraction form for π . Write the formula first, then calculate the answers. Round your answers to the hundredths.

1.



2.

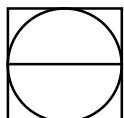


3. Would the answers be the same if you used 3.14 for π for the circles above? Do the calculations.

4. A rabbit hopped 80 feet around a circular flower garden. How far would the rabbit hop if she went straight through the middle of the garden?

5. The diameter of the earth at the equator is 12,760 km. What is the distance around the earth at the equator?

6. A square has a perimeter of 18 cm. What is the circumference of the largest circle that will fit inside the square?



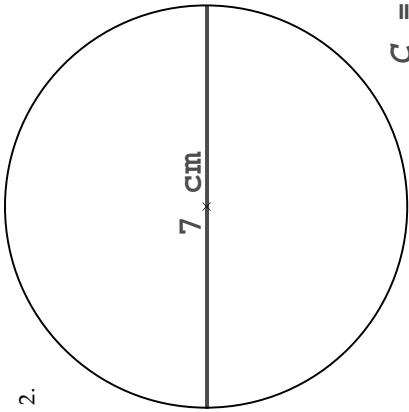
NOTES: If an answer is just written down without a formula, whether it be right or wrong, there is no way to see the steps that procured that answer. If the answer is wrong and there is no written work to follow, the entire process will need to be redone. If there is written work, it is much quicker to see where the error occurred and make corrections. Sometimes the process is accurate, but the arithmetic is wrong.

Some answers may vary slightly due to rounding.

The \approx symbol means approximately. Because we are not using the exact value of pi in our calculations, the symbol \approx needs to be used. That said, if the student uses an equal sign, it is commonly accepted as accurate.

Draw and measure the diameters of the following circles and calculate the circumferences in cm. Use the fraction form for π . Write the formula first, then calculate the answers. Round your answers to the hundredths.

2.

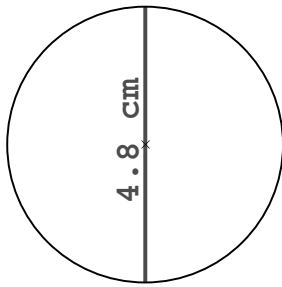


$$C = \pi D$$

$$\approx \frac{22}{7} \times 7$$

$$C \approx 22 \text{ cm}$$

1.



$$C = \pi D$$

$$\approx \frac{22}{7} \times 4.8$$

$$C \approx 15.09 \text{ cm}$$

3. Would the answers be the same if you used 3.14 for π for the circles above? Do the calculations.

$$C = \pi D$$

$$\approx 3.14 \times 4.8$$

$$C \approx 15.07 \text{ cm}$$

$$C = \pi D$$

$$\approx 3.14 \times 7$$

$$C \approx 21.98 \text{ cm}$$

Yes, they are very close to the same.

4. A rabbit hopped 80 feet around a circular flower garden. How far would the rabbit hop if she went straight through the middle of the garden?



$$C = \pi D$$

$$80 \approx 3.14 \times D$$

$$D \approx 80 \div 3.14$$

$$D \approx 25.5 \text{ feet}$$

5. The diameter of the earth at the equator is 12,760 km. What is the distance around the earth at the equator?

$$C = \pi D$$

$$\approx 3.14 \times 12,760$$

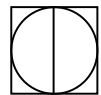
$$C \approx 40,066 \text{ km}$$

$$C = \pi D$$

$$\approx \frac{22}{7} \times 12,760$$

$$C \approx 40,103 \text{ km}$$

6. A square has a perimeter of 18 cm. What is the circumference of the largest circle that will fit inside the square?



$$\text{Side of square} = 18 \div 4 = 4.5 \text{ cm}$$

$$D = 4.5 \text{ cm}$$

$$C = \pi D$$

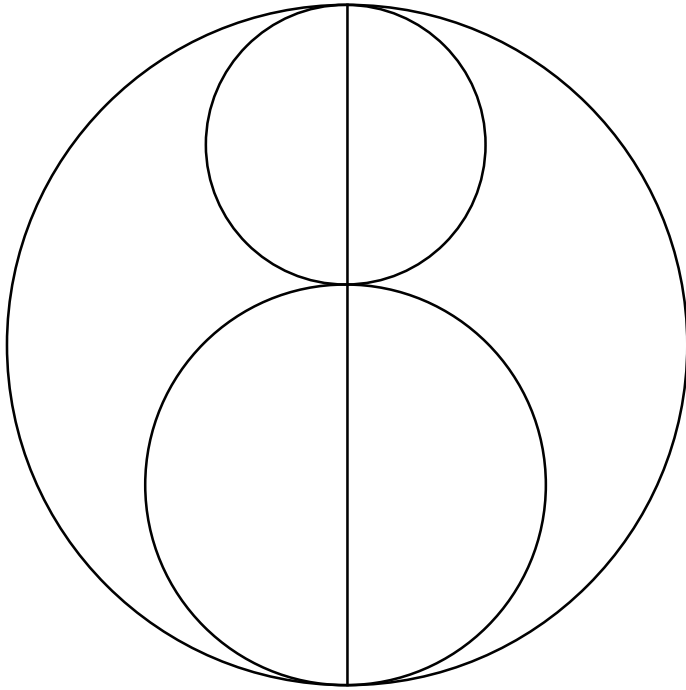
$$\approx 3.14 \times 4.5 \text{ cm}$$

$$C \approx 14.1 \text{ cm}$$

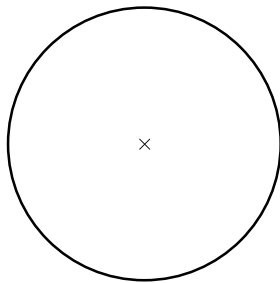
Name: _____

Date: _____

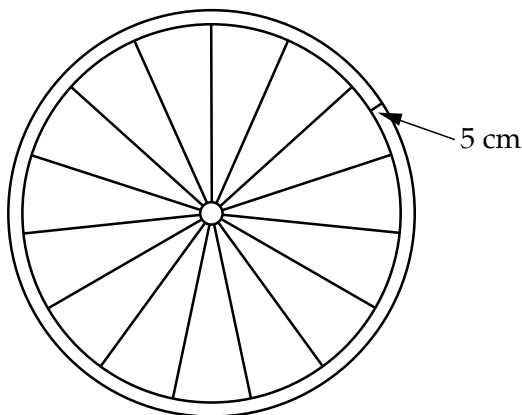
7. Measure the diameters of the three circles in centimeters. Find the circumferences, rounding to tenths. Compare the circumference of the large circle with the circumferences of the two smaller circles.



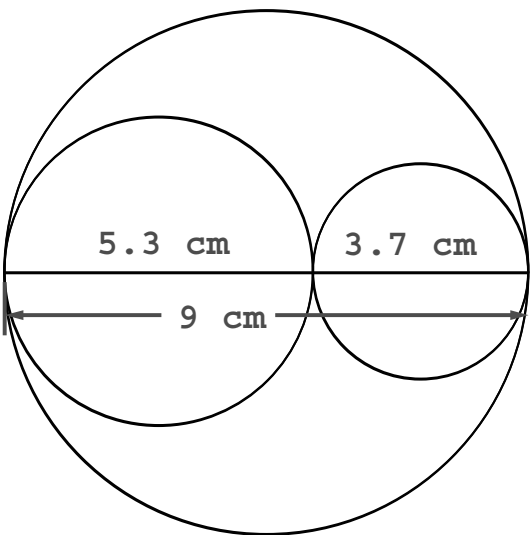
8. Draw and measure the radius of the following circle. Then find the circumference.



9. The circumference of the rim of the bicycle tire (inner circle) is 408 cm and the thickness of the tire wall is 5 cm. What is the circumference of the tire (larger circle)?



7. Measure the diameters of the three circles in centimeters. Find the circumferences, rounding to tenths. Compare the circumference of the large circle with the circumferences of the two smaller circles.



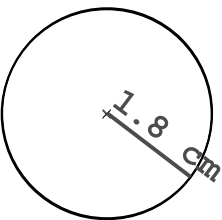
$$\begin{aligned} C(lg) &= \pi D \\ C(lg) &\approx 3.14 \times 9 \\ C(lg) &\approx 28.3 \text{ cm} \end{aligned}$$

$$\begin{aligned} C(\text{med}) &= \pi D \\ C(\text{med}) &\approx 3.14 \times 5.3 \\ C(\text{med}) &\approx 16.6 \text{ cm} \end{aligned}$$

$$\begin{aligned} C(\text{sm}) &= \pi D \\ C(\text{sm}) &\approx 3.14 \times 3.7 \\ C(\text{sm}) &\approx 11.6 \text{ cm} \end{aligned}$$

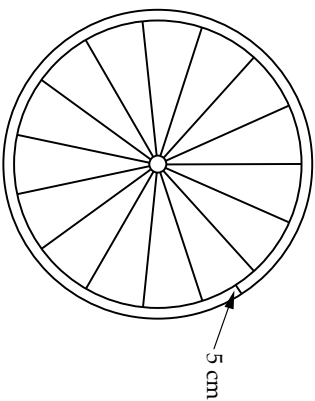
Circumference of
large circle =
circumference of
small and med circles.

8. Draw and measure the radius of the following circle. Then find the circumference.



$$\begin{aligned} C &= 2\pi r \\ &\approx 2 \times 3.14 \times 1.8 \\ C &\approx 11.3 \text{ cm} \end{aligned}$$

9. The circumference of the rim of the bicycle tire (inner circle) is 408 cm and the thickness of the tire wall is 5 cm. What is the circumference of the tire (larger circle)?



$$\begin{aligned} C(rim) &= 2\pi r \\ 408 &\approx 2 \times 3.14 \times r \\ 408 \div (2 \times 3.14) &\approx r(rim) \\ r(rim) &\approx 65 \text{ cm} \end{aligned}$$

$$\begin{aligned} r(\text{tire}) &\approx 65 + 5 \approx 70 \text{ cm} \\ C(\text{tire}) &= 2\pi r \\ &\approx 2 \times 3.14 \times 70 \\ &\approx 440 \text{ cm} \end{aligned}$$

NOTES: Many students (and adults) are surprised that the circumference of the large circle is the same as the two smaller circles: $28.3 = 16.7 + 11.6$.

When doing Problem 8, make sure the student is aware of the difference between diameter and radius. If there is some confusion, have them check their dictionary and/or current lesson.

Problem 9 can be approached using diameter rather than radius. However, using diameter presents an interesting twist in the process.

$$\begin{aligned} C(\text{sm}) &= \pi D \\ 408 &= 3.14 \times D \\ D(\text{sm}) &\approx 408 \div 3.14 \\ D(\text{sm}) &\approx 130 \text{ cm} \end{aligned}$$

$$D(lg) \approx 130 + 5 + 5$$

The thickness of the tire needs to be added to both sides of the circumference, not just one side. This is a common error.

$$\begin{aligned} D(lg) &\approx 140 \text{ cm} \\ C(lg) &= \pi D \\ &\approx 3.14 \times 140 \\ &\approx 440 \text{ cm} \end{aligned}$$

DICTIONARY TERMS: pi, π , \approx

LESSON 121: LINES OF SYMMETRY

OBJECTIVES:

1. To learn the terms *line of symmetry* and *ellipse*
2. To compare a line of symmetry to a line of reflection
3. To find the lines of symmetry in a figure
4. To learn the terms *maximum* and *minimum*
5. To learn about the infinity symbol, “ ∞ ”

MATERIALS:

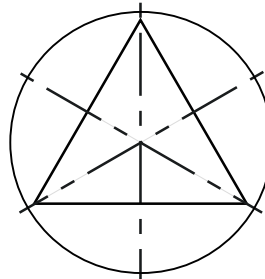
1. Math Dictionary
2. Worksheets 121-1 and 121-2, Lines of Symmetry
3. Drawing board, T-square, and triangles
4. **Geometry reflector, optional**
5. *Math Card Games* book

ACTIVITIES:

Line of symmetry. A *line of symmetry* divides a figure into two parts with one part being the reflection, or mirror image, of the other part.

A good way to check the line of symmetry is to fold the figure in half. If the two halves match, the fold line is the line of symmetry.

Line symmetry is very common in nature, art, and logos (symbols identifying businesses or brands). In the examples below, notice that the line of symmetry may be at any angle. Also see that a figure can have more than one line of symmetry.



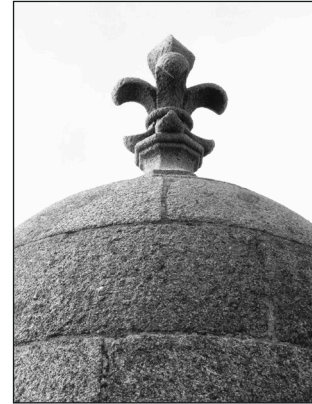
Comparing to a line of reflection. Reflections are a transformation, which means an object is transformed, or changed, into something else.

To think how a line of reflection is related to a line of symmetry, take a figure with a line of symmetry and remove or cover half of it along that line. Then reflect the remaining part of the figure about the line of symmetry, which is now the line of reflection. What do you notice? You're back to the original object!

Therefore, a line of symmetry is a line within a figure. A line of reflection is usually outside the figure.

Some face fun. Faces are almost symmetric. It is fun to make a picture of a person or an animal completely symmetric. To do this, first put the picture into computer software, either Adobe Photoshop or another similar application. Remove half. Copy the remaining half and reflect it horizontally. Lastly, place the reflected image along side the original half. See an example on the next page.

EXTRAS:



The Fleur de Lise, an important symbol in France.

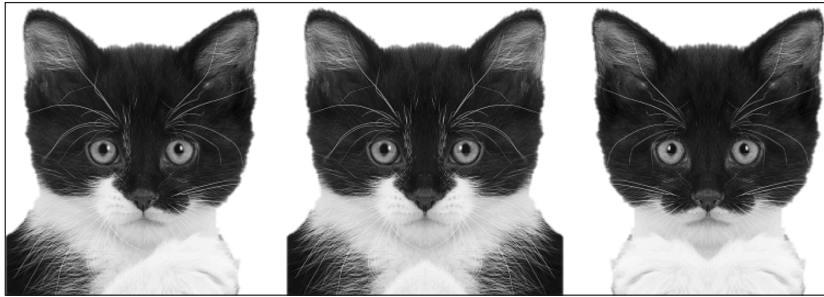


Building in Maryland.

Remember lines of reflection are indicated with the special dashed line, a long dash alternating with short dash. Lines of symmetry are also indicated with the same special dashed line.

LESSON CONTINUES ON THE NEXT PAGE.

ACTIVITIES:



A kitten.

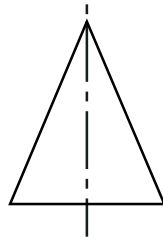
Kitten with the left side reflected.

Kitten with the right side reflected.

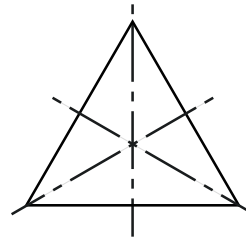
Worksheets 121-1 and 121-2. Complete the first 19 questions on the worksheets now.

Maximum and minimum. You may have seen the terms *maximum* and *minimum* on highway signs. The maximum speed is the highest speed a person may legally drive. The minimum speed is the lowest speed allowed on the road. So, the maximum is the greatest and the minimum is the least.

Let's look at an isosceles triangle as an example for the maximum and minimum number of lines of symmetry. The left figure below shows the usual isosceles triangle. Clearly, it has one line of symmetry because an isosceles triangle has two equal sides.



How about the equilateral triangle shown on the right with its three lines of symmetry? Doesn't it also have two equal sides? Yes!



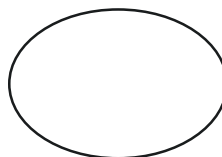
An equilateral triangle is a special case of an isosceles triangle. Therefore, the minimum number of lines of symmetry in an isosceles triangle is 1 and the maximum number is 3.

Infinity symbol. Infinity is not a number, but a concept. For example, how many numbers are there? We say the answer is infinite because whatever number you say, I can say one higher. There is no limit!

You will need to use the infinity symbol, " ∞ ", for the table in problem 20. Sometimes the symbol is called a "lazy eight."

Worksheet 121-2, problem 20. You may need to look up some of the definitions in your Math Dictionary to answer this problem. Remember the difference between a polygon and a regular polygon.

Sometimes people will call a squashed circle an oval. The mathematical name is *ellipse*. See the figure on the right. A circle is a special ellipse in the same way an equilateral triangle is a special isosceles triangle.



Today's game. Play your choice of math card game from the *Math Card Games* book.

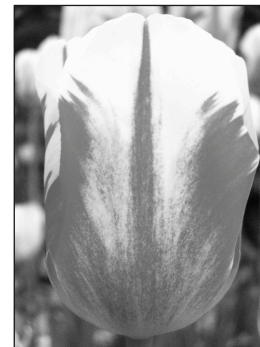
EXTRAS:



From the botanical garden in Big Island, Hawaii.

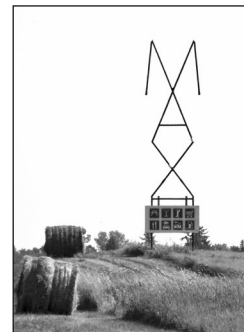


Road sign showing maximum and minimum.



A tulip from Monet's Gardens in Giverny, France.

To write the infinity symbol, " ∞ ", start in the middle, draw a loop on one side and then the other side.

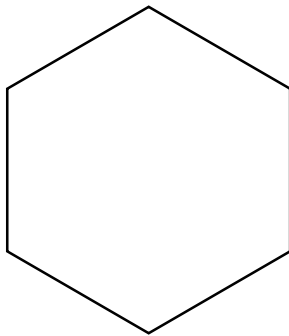
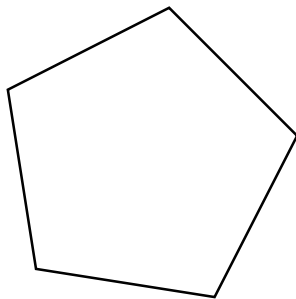
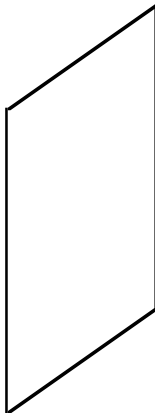
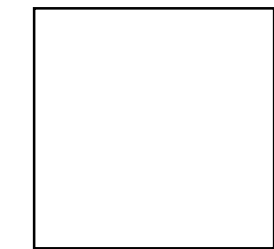


Sign near Max, ND. The line of reflection is easy to see.

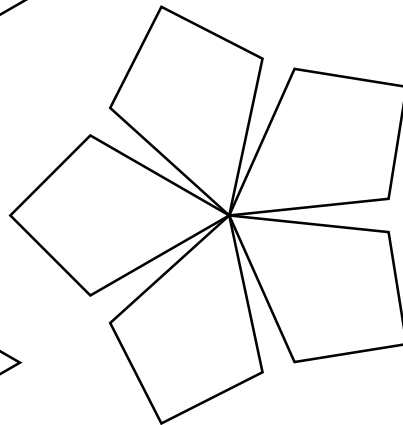
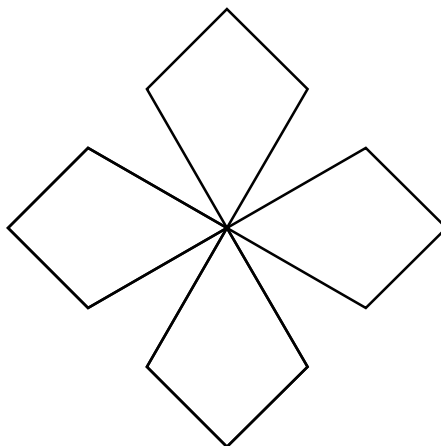
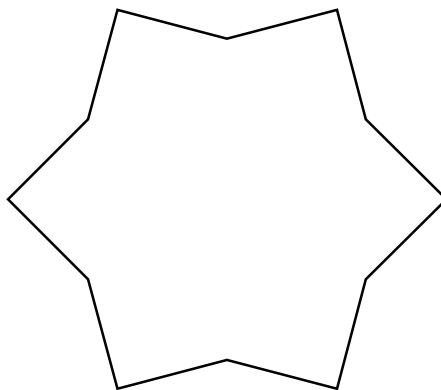
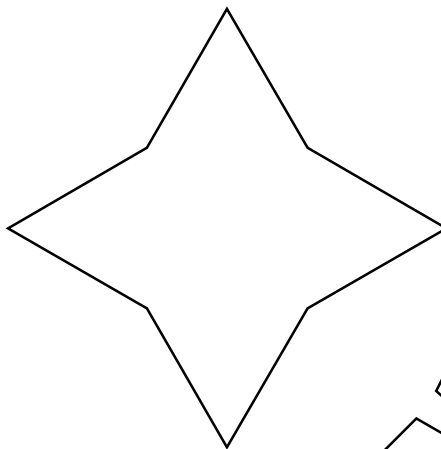
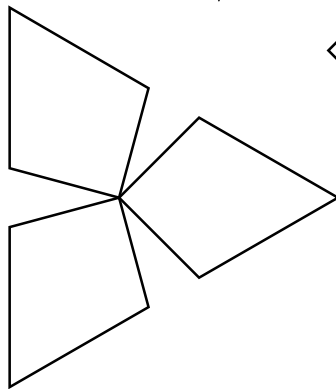
Name: _____

Date: _____

1–6. Draw the lines of symmetry in the following figures. There are 19 total lines. Use your drawing tools where possible.

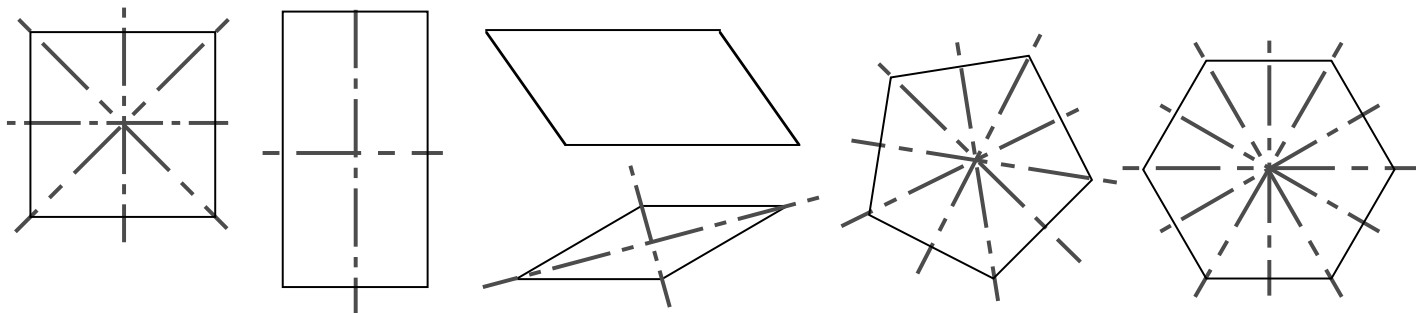


7–11. Use your drawing tools, where possible, to draw the lines of symmetry. There are 22 such lines on these five figures.

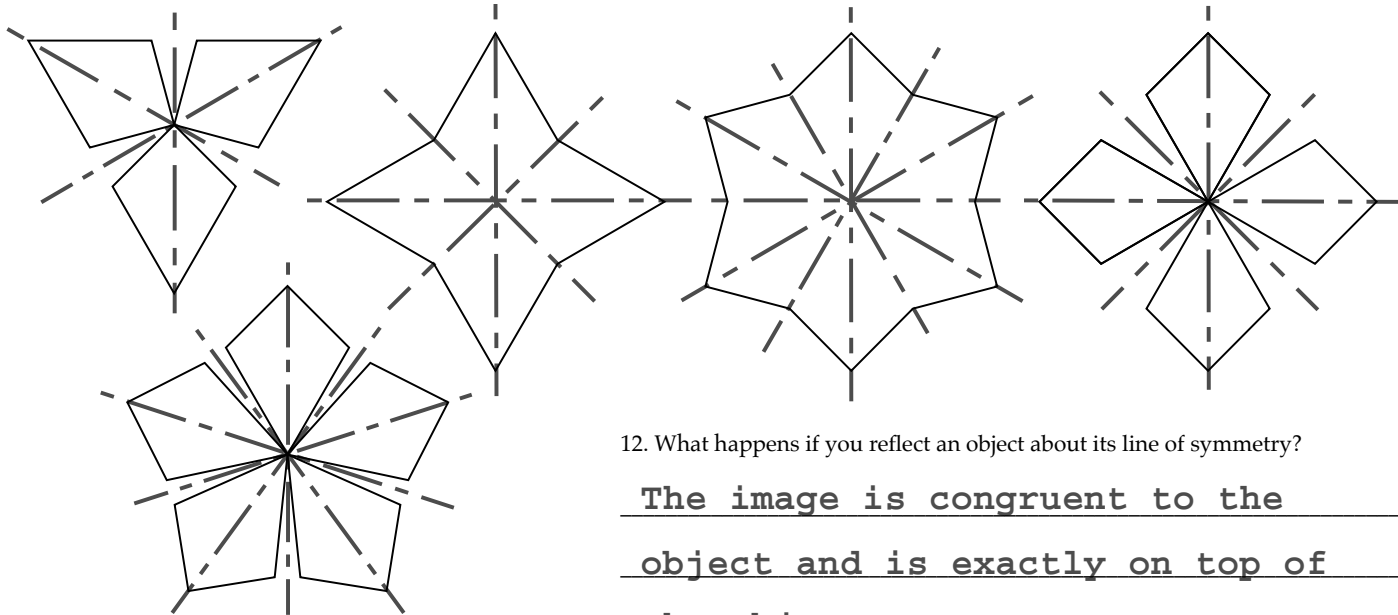


12. What happens if you reflect an object about its line of symmetry?

1–6. Draw the lines of symmetry in the following figures. There are 19 total lines. Use your drawing tools where possible.



7–11. Use your drawing tools, where possible, to draw the lines of symmetry. There are 22 such lines on these five figures.



12. What happens if you reflect an object about its line of symmetry?

The image is congruent to the
object and is exactly on top of
the object.

NOTES: The lines of symmetry are indicated with a long dash alternating with a short dash. If the student uses a solid line, it is not accurate, but it is acceptable.

Name: _____

Date: _____

20. For the following figures, find the minimum and maximum number of lines of symmetry. Draw the figures if it helps you.

Figure	Minimum Lines of Symmetry	Maximum Lines of Symmetry
Quadrilateral		
Rectangle		
Square		
Rhombus		
Parallelogram		
Trapezoid		
Equilateral Triangle		
Scalene Triangle		
Right Triangle		
Pentagon		
Hexagon		
Octagon		
Circle		
Ellipse		

13. List the 16 capital letters that have line symmetry.

14. List the 4 digits that have line symmetry.

15–18. Draw the line of symmetry through the following words.

MOM

LOF

MATH

CODEBOOK

19. Using capital letters write the states, Iowa, Hawaii, and Ohio, with line symmetry. Draw the lines of symmetry.

CONTINUE READING THE LESSON.

13. List the 16 capital letters that have line symmetry.

A B C D E H I K M O T U V W X Y

14. List the 4 digits that have line symmetry.

0 1 3 8

15–18. Draw the line of symmetry through the following words.

MOM

TOT

MATH

CODEBOOK

19. Using capital letters write the states, Iowa, Hawaii, and Ohio, with line symmetry. Draw the lines of symmetry.

IOWA

HAWAII

OHIO

or

OHIO

CONTINUE READING THE LESSON.

20. For the following figures, find the minimum and maximum number of lines of symmetry. Draw the figures if it helps you.





























Figure	Minimum Lines of Symmetry	Maximum Lines of Symmetry
Quadrilateral	0	4
Rectangle	2	4
Square	4	4
Rhombus	2	4
Parallelogram	0	4
Trapezoid	0	1
Equilateral Triangle	3	3
Scalene Triangle	0	0
Right Triangle	0	1
Pentagon	0	5
Hexagon	0	6
Octagon	0	8
Circle	∞	∞
Ellipse	2	∞

NOTES: There may be some variances with Problem 13 due to the student's formation of the alphabet. For example, does the capital letter U have a "tail" on the right lower edge or not? Does the letter Y have the bottom half diagonal or centered?

Technically, given the font used, CODEBOOK does not have a line of symmetry because of the letter K. That said, the general consensus for our purposes is that it does have a line of symmetry.

Problem 20 uses the definitions of various figures. Here is the same chart with possible images included.

DICTIONARY TERMS: lines of symmetry, ellipse, maximum, minimum, infinity symbol "∞".

Figure	Minimum Lines of Symmetry	Maximum Lines of Symmetry
Quadrilateral	 0	 4
Rectangle	 2	 4
Square	 4	 4
Rhombus	 2	 4
Parallelogram	 0	 4
Trapezoid	 0	 1
Equilateral Triangle	 3	 3
Scalene Triangle	 0	 0
Right Triangle	 0	 1
Pentagon	 0	 5
Hexagon	 0	 6
Octagon	 0	 8
Circle	 ∞	 ∞
Ellipse	 2	 ∞