RightStart™ Mathematics

by Activities for Learning, Inc.

Dear Educator,

The following are some of the principles that make the RightStart™ approach different from traditional primary mathematics as taught in the U.S.

• **Minimizing counting.** We know from research that 5-month-old babies are able to add and subtract up to 3. This they do, not by counting, but by visualizing quantities. Japanese teachers believe that rote counting does not further a child's mathematical ability. In other words, counting to 100 is no more helpful in learning math than reciting the alphabet helps in learning to read. In RightStart™, counting is minimized, and visualization is emphasized.

• **Grouping in fives.** It is relatively easy to detect up to five objects—five can be distinguished from four because it has a middle while four does not. Beyond five, very few people can identify or visualize objects. Thus, the Romans grouped their numerals in fives; consider VIII (8). Orchestral arrangers grouped the 10 lines of music into two staffs with five lines. RightStart™ groups in fives and tens. Strategies to learn the facts are built on this grouping.

• **Working with the Cotter Abacus (AL Abacus).** To continue developing the visualization skills they possessed as infants, children use the Cotter Abacus. The beads are grouped in fives to allow quick recognition and subsequent visualization. The reverse side teaches trading in the thousands. To learn their facts, the Cotter Abacus provides children with visual strategies. Children enjoy using the Cotter Abacus, but it doesn't become a crutch. Five year-old Stan was asked, "How much is 11 + 6?" He said 17. Then, he replied, "I've got the abacus in my mind."

• **Playing games.** Flash cards are not part of RightStart™. Flash cards and timed tests come with a tremendous cost: the stress takes the joy out of learning mathematics. We have millions of people in this country who avoid math whenever possible; many have said that is the reason. Instead, the children using RightStart™ play games to become fluent with their facts and computation. Parents are encouraged to play games with their child.

• **Naming numbers explicitly.** In the U.S. children speaking English experience considerable difficulty learning place value. Indeed, only half of them master it by the end of the fourth grade. On the other hand, Asian children master place value in the first grade. Asian languages support this understanding through explicit number naming. For example, numbers 11 to 13 are called "ten-1, ten-2, ten-3" and 20 to 22 are "2 ten, 2 ten-1, 2 ten-2." RightStart™ introduces the explicit number naming in the early grades, then transitions to traditional names.

• **Introducing thousands in the first grade.** To understand the never-ending pattern that ten ones equals 10, ten tens equals 100, ten hundreds equals 1000, and so forth, children must work with thousands. This gives the child the whole picture before working with details. In first grade with RightStart™, the children learn to add four-digit numbers with trading early.

• **Overlapping place-value cards.** Children have difficulty with the concept that the 3 in 32 actually means 3 tens. RightStart™ uses place-value cards. For example, the card with "30" is read 3-ten. To build 32, the child places the 2-card on the 0 of the 30-card, forming 32. Note these cards encourage reading numbers in the normal left to right order.

• **Computing mentally.** Most people when adding 24 + 38 compute it mentally, rather than resorting to paper and pencil or a calculator. Therefore, in RightStart™, first graders learn to add two-digit numbers mentally. They use the efficient method of starting at the left.

• **Learning fractions with a linear model.** The linear model gives children an overview of fractions and allows them to see the relationship between fractions and allows understanding of fractions greater than one.

• **Using correct vocabulary.** RightStart™ stresses correct terminology. Equation, which indicates equality, is used rather than "number sentence." The phrase "take away" is avoided because it limits students' understanding of subtraction—sometimes subtraction is going up, as in making change. Trading is used instead of "regrouping" because the latter does not imply equality to children as does trading.

Helping students understand, apply, and enjoy mathematics.

How This Program Was Developed

We have been hearing for years that Japanese students do better than U.S. students in math in Japan. The Asian students are ahead by the middle of first grade. And the gap widens every year thereafter.

Many explanations have been given, including less diversity and a longer school year. Japanese students attend school 240 days a year.

A third explanation given is that the Asian public values and supports education more than we do. A first grade teacher has the same status as a university professor. If a student falls behind, the family, not the school, helps the child or hires a tutor. Students often attend after-school classes.

A fourth explanation involves the philosophy of learning. Asians and Europeans believe anyone can learn mathematics or even play the violin. It is not a matter of talent, but of good teaching and hard work.

Although these explanations are valid, I decided to take a careful look at how mathematics is taught in Japanese first grades. Japan has a national curriculum, so there is little variation among teachers.

I found some important differences. One of these is the way the Asians name their numbers. In English we count ten, eleven, twelve, thirteen, and so on, which doesn't give the child a clue about tens and ones. But in Asian languages, one counts by saying ten-1, ten-2, ten-3 for the teens, and 2-ten 1, 2-ten 2, and 2-ten 3 for the twenties.

Still another difference is their criteria for manipulatives. Americans think the more the better. Asians prefer very few, but insist that they be imaginable, that is, visualizable. That is one reason they do not use colored rods. You can imagine the one and the three, but try imagining a brown eight–the quantity eight, not the color. It cannot be done without grouping.

Another important difference is the emphasis on non-counting strategies for computation. Japanese children are discouraged from counting; rather they are taught to see quantities in groups of fives and tens.

For example, when an American child wants to know 9 + 4, most likely the child will start with 9 and count up 4. In contrast, the Asian child will think that if he takes 1 from the 4 and puts it with the 9, then he will have 10 and 3, or 13. Unfortunately, very few American first-graders at the end of the year even know that 10 + 3 is 13.

I decided to conduct research using some of these ideas in two similar first grade classrooms. The control group studied math in the traditional workbook-based manner. The other class used the lesson plans I developed. The children used that special number naming for three months.

They also used a special abacus I designed, based on fives and tens. I asked 5-year-old Stan how much is 11 + 6. Then I asked him how he knew. He replied, "I have the abacus in my mind."

The children were working with thousands by the sixth week. They figured out how to add 4-digit numbers on paper after learning how on the abacus.

Every child in the experimental class, including those enrolled in special education classes, could add numbers like 9 + 4, by changing it to 10 + 3.

I asked the children to explain what the 6 and 2 mean in the number 26. Ninety-three percent of the children in the experimental group explained it correctly while only 50% of third graders did so in another study.

I gave the children some base ten rods (none of them had seen them before) that looked like ones and tens and asked them to make 48. Then I asked them to subtract 14. The children in the control group counted 14 ones, while the experimental class removed 1 ten and 4 ones. This indicated that they saw 14 as 1 ten and 4 ones and not as 14 ones. This view of numbers is vital to understanding algorithms, or procedures, for doing arithmetic.

I asked the experimental class to mentally add 64 + 20, which only 52% of nine-year-olds on the 1986 National test did correctly; 56% of those in the experimental class could do it.

Since children often confuse columns when taught traditionally, I wrote 2304 + 86 = horizontally and asked them to find the sum any way they liked. Fifty-six percent did so correctly, including one child who did it in his head.

The following year I revised the lesson plans and both first grade classes used these methods. I am delighted to report that on a national standardized test, both classes scored at the 98th percentile.

Joan A. Cotter, Ph.D.

Some General Thoughts on Teaching Mathematics

1. Only five percent of mathematics should be learned by rote; 95 percent should be understood.
2. Real learning builds on what the child already knows. Rote teaching ignores it.
3. Contrary to the common myth, "young children can think both concretely and abstractly. Development is not a kind of inevitable unfolding in which one simply waits until a child is cognitively 'ready.'" —*Foundations for Success* NMAP
4. What is developmentally appropriate is not a simple function of age or grade, but rather is largely contingent on prior opportunities to learn." —Duschl & others
5. Understanding a new model is easier if you have made one yourself. So, a child needs to construct a graph before attempting to read a ready-made graph.
6. Good manipulatives cause confusion at first. If a new manipulative makes perfect sense at first sight, it is not needed. Trying to understand and relate it to previous knowledge is what leads to greater learning. —Richard Behr & others.
7. According to Arthur Baroody, "Teaching mathematics is essentially a process of translating mathematics into a form children can comprehend, providing experiences that enable children to discover relationships and construct meanings, and creating opportunities to develop and exercise mathematical reasoning."
8. Lauren Resnick says, "Good mathematics learners expect to be able to make sense out of rules they are taught, and they apply some energy and time to the task of making sense. By contrast, those less adept in mathematics try to memorize and apply the rules that are taught, but do not attempt to relate these rules to what they know about mathematics at a more intuitive level."
9. Mindy Holte puts learning the facts in proper perspective when she says, "In our concern about the memorization of math facts or solving problems, we must not forget that the root of mathematical study is the creation of mental pictures in the imagination and manipulating those images and relationships using the power of reason and logic." She also emphasizes the ability to imagine or visualize, an important skill in mathematics and other areas.
10. The only students who like flash cards are those who do not need them.
11. Mathematics is not a solitary pursuit. According to Richard Skemp, solitary math on paper is like reading music, rather than listening to it: "Mathematics, like music, needs to be expressed in physical actions and human interactions before its symbols can evoke the silent patterns of mathematical ideas (like musical notes), simultaneous relationships (like harmonies) and expositions or proofs (like melodies)."
12. "More than most other school subjects, mathematics offers special opportunities for children to learn the power of thought as distinct from the power of authority. This is a very important lesson to learn, an essential step in the emergence of independent thinking." —*Everybody Counts*

13. The role of the teacher is to encourage thinking by asking questions, not giving answers. Once you give an answer, thinking usually stops.
14. Putting thoughts into words helps the learning process.
15. Help the children realize that it is their responsibility to ask questions when they do not understand. Do not settle for "I don't get it."
16. The difference between a novice and an expert is that an expert catches errors much more quickly. A violinist adjusts pitch so quickly that the audience does not hear it.
17. Europeans and Asians believe learning occurs not because of ability, but primarily because of effort. In the ability model of learning, errors are a sign of failure. In the effort model, errors are natural. In Japanese classrooms, the teachers discuss errors with the whole class.
18. For teaching vocabulary, be sure either the word or the concept is known. For example, if a child is familiar with six-sided figures, we can give him the word, hexagon. Or, if he has heard the word, multiply, we can tell him what it means. It is difficult to learn a new concept and the term simultaneously.
19. Introduce new concepts globally before details. This lets the children know where they are headed.
20. Informal mathematics should precede paper and pencil work. Long before a child learns how to add fractions with unlike denominators, she should be able to add one half and one fourth mentally.
21. Some pairs of concepts are easier to remember if one of them is thought of as dominant. Then the non-dominant concept is simply the other one. For example, if even is dominant over odd, an odd number is one that is not even.
22. Worksheets should also make the child think. Therefore, they should not be a large collection of similar exercises, but should present a variety. In RightStart™ Mathematics, they are designed to be done independently.
23. Keep math time enjoyable. Our emotional state at the time we learn something is attached to that information. A person who dislikes math will avoid it and a child under stress stops learning. If a lesson is too hard, stop and play a game. Try the lesson again later.
24. In Japan students spend more time on fewer problems. Teachers do not concern themselves with attention spans as is done in the U.S.
25. In Japan the goal of the math lesson is that the student has understood a concept, not necessarily has done something (a worksheet).
26. The calendar must show the entire month, so the children can plan ahead. The days passed can be crossed out or the current day circled.
27. A real mathematical problem is one in which the procedures to find the answer are not obvious. It is like a puzzle, needing trial and error. Emphasize the satisfaction of solving problems and like puzzles, of not giving away the solution to others.

RightStart™ Mathematics

Ten major characteristics make this research-based program effective:

1. Refers to quantities of up to 5 as a group; discourages counting individually. Uses fingers and tally sticks to show quantities up to 10; teaches quantities 6 to 10 as 5 plus a quantity, for example 6 = 5 + 1.
2. Avoids counting procedures for finding sums and differences. Teaches five- and ten-based strategies for the facts that are both visual and visualizable.
3. Employs games, not flash cards, for practice.
4. Once quantities 1 to 10 are known, proceeds to 10 as a unit. Temporarily uses the "math way" of naming numbers; for example, "1 ten-1" (or "ten-1") for eleven, "1-ten 2" for twelve, "2-ten" for twenty, and "2-ten 5" for twenty-five.
5. Uses expanded notation (overlapping) place-value cards for recording tens and ones; the ones card is placed on the zero of the tens card. Encourages a child to read numbers starting at the left and not backward by starting at the ones.
6. Proceeds rapidly to hundreds and thousands using manipulatives and place-value cards. Provides opportunities for trading between ones and tens, tens and hundreds, and hundreds and thousands with manipulatives.
7. Teaches mental computation. Investigates informal solutions, often through story problems, before learning procedures.
8. Teaches four-digit addition on the abacus, letting the child discover the paper and pencil algorithm.
9. Introduces fractions with a linear visual model, including all fractions from 1⁄2 to 1⁄10. "Pies" are not used initially because they cannot show fractions greater than 1. Later, the tenths will become the basis for decimals.
10. Teaches short division (where only the answer is written down) for single-digit divisors, before long division.

Second Edition

Many changes have occurred since the first RightStart™ lessons were begun in 1994. First, mathematics is used more widely in many fields, for example, architecture, science, technology, and medicine. Today, many careers require math beyond basic arithmetic. Second, research has given us new insights into how children learn mathematics. Third, kindergarten has become much more academic, and fourth, most children are tested to ensure their preparedness for the next step.

This second edition is updated to reflect new research and applications. RightStart™ Mathematics Second Edition, incorporates the Common Core State Standards as the basic minimum. Topics within each grade level are always taught with the most appropriate method using the best approach with the child and teacher in mind.

Daily Lessons

Objectives. The objectives outline the purpose and goal of the lesson. Some possibilities are to introduce, to build, to learn a term, to practice, or to review.

Materials. The Math Set of manipulatives includes the specially crafted items needed to teach RightStart™ Mathematics. Occasionally, common objects such as scissors will be needed. These items are indicated by boldface type.

Warm-up. The warm-up time is the time for quick review, memory work, and sometimes an introduction to the day's topics. The dry erase board makes an ideal slate for quick responses.

Activities. The Activities for Teaching section is the heart of the lesson; it starts on the left page and continues to the right page. These are the instructions for teaching the lesson. The expected answers from the child are given in square brackets.

Establish with the children some indication when you want a quick response and when you want a more thoughtful response. Research shows that the quiet time for thoughtful response should be about three seconds. Avoid talking during this quiet time; resist the temptation to rephrase the question. This quiet time gives the slower child time to think and the quicker child time to think more deeply.

Encourage the child to develop persistence and perseverance. Avoid giving hints or explanations too quickly. Children tend to stop thinking once they hear the answer.

Explanations. Special background notes for the teacher are given in Explanations.

Worksheets. The worksheets are designed to give the children a chance to think about and to practice the day's lesson. The children are to do them independently. Some lessons, especially in the early grades, have no worksheet.

Games. Games, not worksheets or flash cards, provide practice. The games, found in the *Math Card Games* book, can be played as many times as necessary until proficiency or memorization takes place. They are as important to learning math as books are to reading. The *Math Card Games* book also includes extra games for the child needing more help, and some more challenging games for the advanced child.

In conclusion. Each lesson ends with a short summary called, "In conclusion," where the child answers a few short questions based on the day's learning.

Number of lessons. Generally, each lesson is to be done in one day and each manual, in one school year. Complete each manual before going on to the next grade.

Comments. We really want to hear how this program is working. Please let us know any improvements and suggestions that you may have.

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