

Teaching Primary Mathematics without Counting and Place Value with Transparent Number Naming

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The Mathematics Education for the Future Project

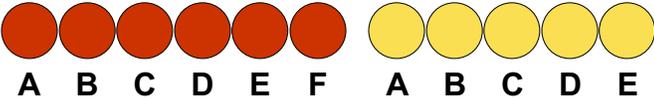
King's College, Cambridge

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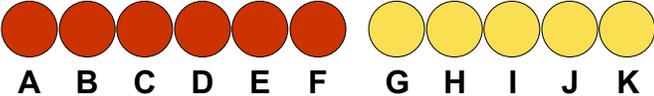
Counting Model

From a child's perspective

$$F + E =$$



What is the sum?



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Subitizing: Counting Alternative

Grouping in fives



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Subitizing: Counting Alternative

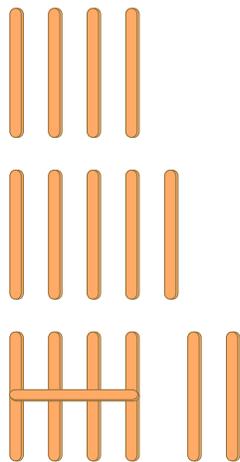
- Subitizing “allows the child to grasp the whole and the elements at the same time.” *-Benoit*
- Subitizing seems to be a necessary skill for understanding what the counting process means. *-Glaserfeld*
- Children who can subitize perform better in mathematics long term. *-Butterworth*
- Counting-on is a difficult skill for many children.
- Math anxiety affects counting ability, but *not* subitizing ability.

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Learning 1 to 10

Using tally sticks



Tally sticks

	1			6
	2			7
	3			8
	4			9
	5			10

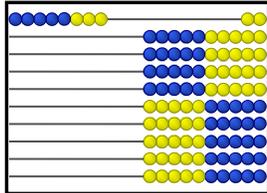
Number Chart

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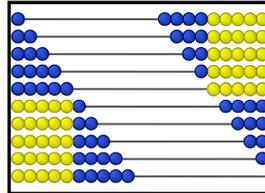
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Learning 1 to 10

Using the AL Abacus

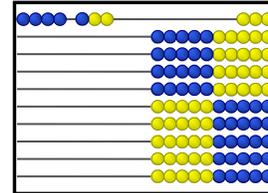


Entering 8



The stairs

Adding



$4 + 3 = 7$

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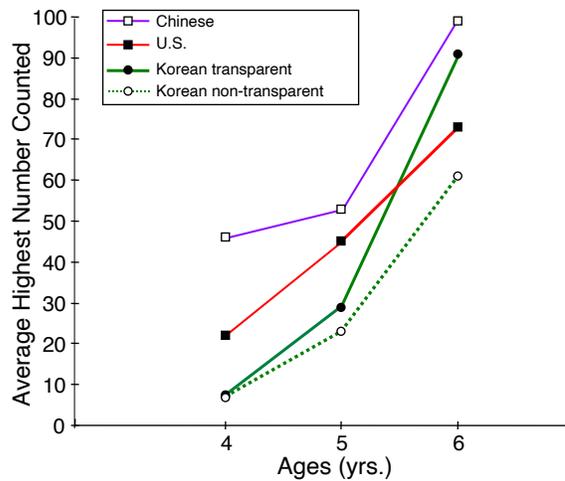
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Transparent Number Naming

11 = ten 1
 12 = ten 2
 13 = ten 3
 14 = ten 4

 19 = ten 9
 20 = 2-ten
 21 = 2-ten 1
 22 = 2-ten 2
 23 = 2-ten 3

 99 = 9-ten 9



Song, M., & Ginsburg, H. (1988). p. 326. The effect of the Korean number system on young children's counting: A natural experiment in numerical bilingualism. *International Journal of Psychology*, 23, 319-332.

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Transparent Number Naming

Some salient features

- Only 11 words are needed to count to 100 with transparent number naming, 28 in English.
- Asian children learn math with transparent number names. They understand place value very young.
- Mathematics is the science of patterns. Number names must be an example.
- A child who is hearing impaired often cannot distinguish 14 from 40 or 13 from 30.
- Learning two languages helps brain development.

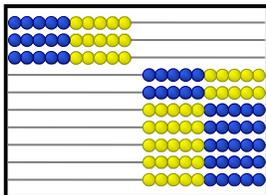
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Transparent Number Naming

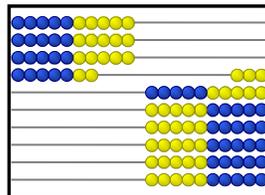
Place-value cards

30



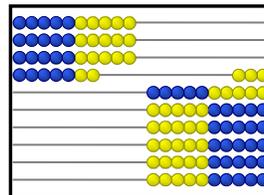
3-ten

30 7 → 307



3-ten 7

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The "0" in 30 is merely hidden.

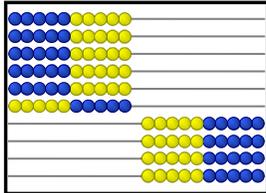
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Transparent Number Naming

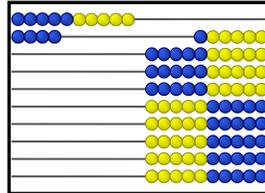
Traditional names

The tens



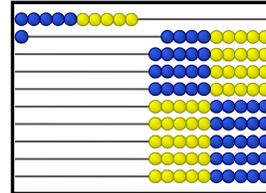
6-tens is also *sixty*.
(-ty means *ten*)

Numbers 13-19



ten 4
↓
teen 4
↙ ↘
fourteen

Numbers 11-12



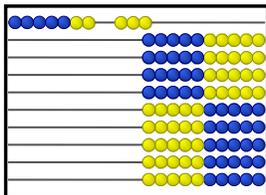
a one left
↙ ↘
a left one
↓
eleven

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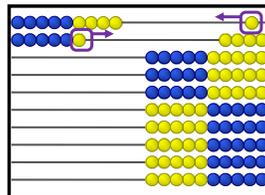
Addition Strategies

Partitioning Ten



$$10 = 7 + \underline{\quad}$$

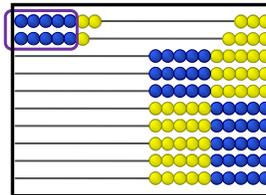
Complete the Ten



$$9 + 6 = \underline{\quad}$$

Take 1 from the
6 and give it to 9.

Two Fives



$$7 + 6 = \underline{\quad}$$

$$10 + 3 = 13$$

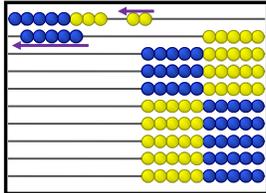
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Subtraction Strategies

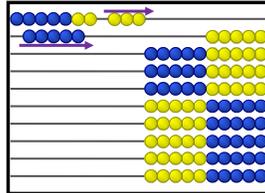
Subtracting from 10 or more

Going Up



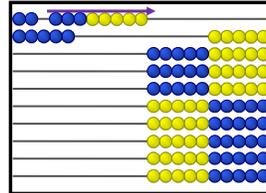
$15 - 8 = \underline{\quad}$
 Start with 8; add
 2 and then 5.
 $2 + 5 = 7$

Part from Ten



$15 - 8 = \underline{\quad}$
 Subtract 5 from
 5 and 3 from 10.
 Difference is 7.

All from Ten



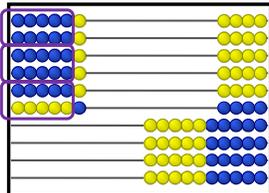
$15 - 8 = \underline{\quad}$
 Subtract 8 from
 10 then add
 2 and 5.

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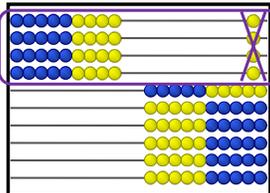
Multiplication Strategies

Example 1



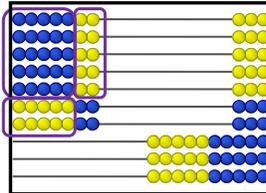
$6 \square 6 =$
 $10 \square 3 + 6 = 36$

Example 2



$9 \square 4 =$
 $40 - 4 = 36$

Example 3



$7 \square 7 = 25 + 10 +$
 $10 + 4 = 49$

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Summary

- Memorizing the counting words to 100 is unnecessary and unproductive.
- Infants naturally subitize and visualize.
- Visualizing requires subitizing in groups of fives.
- Subitizing and visualizing reduce memory load.
- Transparent number naming gives children support in learning place value.
- Place value is the foundation of arithmetic.
- Effective strategies for mastering the facts are visualizable and are based around 5 and 10.