# RightStart™ Tutoring

by Joan A. Cotter, Ph.D.
and Kathleen Cotter Clayton

# FRACTIONS
## IN $42\frac{1}{2}$ DAYS
### LESSONS

AL Activities for Learning, Inc.

A special thank you to Dustin Sailer for his contribution to this project.



Printed in the United States of America

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ISBN 978-1-931980-54-8
June 2021

# INTRODUCTION

This book is based on fractions lessons, activities, and games from the RightStart™ Mathematics curriculum and from Math Card Games, 5th edition, both written by Dr. Joan A. Cotter. It is intended for both students and adults who need and want to understand fractions from the basic understanding to dividing fractions. This manual can be used with any mathematics program; knowledge of the RightStart™ Mathematics program is not needed.

Fractions are introduced with a linear visual model, including all fractions from $\frac{1}{2}$ to $\frac{1}{10}$. Most work with fractions should be done with fraction charts, a copy of which is in the Appendix on page 1. Sturdier charts, wood, plastic, and magnetic, are available from RightStartMath.com. The charts promote investigation using informal solutions before learning procedures. When working with fractions, children should only use the horizontal form, not the slanted line.

The most frequent reason for confusion with fractions is a vague understanding of what fraction are and how they are related to each other. This uncertainty creates an environment of minimal or no understanding while memorizing of the procedures with marginal success. In these $42\frac{1}{2}$ days of lessons and games, we will lay a solid foundation, and proceed step by step to create clear understanding. Asians and Europeans have the philosophy of learning that anyone can learn mathematics, including fractions, or play the violin. It is not a matter of talent, but of good teaching and hard work. This book will provide the teaching guide and makes the work interesting with games and activities. There are 56 games and 17 worksheets over $42\frac{1}{2}$ days of lessons.

Some might feel that fractions are becoming obsolete because the metric system does not need them and calculators use decimals. However, fractions are the culmination of arithmetic and are essential for understanding algebra and other advanced topics. In essence, fractions are division. To be successful with more advanced fractions, students must be thoroughly familiar with multiplication and division of whole numbers. This background is needed to simplify a fraction to lowest terms and to find a lowest common denominator.

We hope that through these lessons and games, students will develop a renewed interest and enjoyment in mathematics, thereby enriching their lives. We also hope some of them will become tomorrow's mathematicians, scientists, and engineers.

We really want to hear how this program is a success for you and your students. Please share your experience and let us know!

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# DAILY LESSONS

## Lightbulb and Target 

This identifies why the day's topic is important and the goal of the lesson.

## You will need

Materials needed for the day's activities will be identified. It might be the fraction chart or fraction pieces, cards, or common objects such as ruler or paper and pencil. If an appendix page or worksheet is needed, it will be listed here.

Manipulatives, such as the fraction chart or multiplication table, are not to be regarded as crutches. They enable the student to build a mental model, necessary for concept formation. In practice, they will refer to them less and less and finally not at all. Let each student decide when they no longer needs them. Sometimes just the security of having them nearby helps, even if they are not looked at. At the right time, a student may respond to the challenge of playing without them.

## Activities

This section is the heart of each day's lesson. These are the instructions for teaching the lesson. The expected answers from the student are given in square brackets.

Research shows that the quiet time for thoughtful response should be about three to five seconds. Avoid talking during this quiet time; resist the temptation to rephrase the question. This time gives the slower student time to think and the quicker student time to think more deeply. Encourage the student to develop persistence and perseverance. Avoid giving hints or explanations too quickly. Students, and people in general, tend to stop thinking once they hear the answer.

## Games

Daily games, not worksheets or flash cards, provide practice of the new skills. The games can be played as many times as necessary until proficiency takes place. They are as important to learning math as books are to reading. Reviewing old games lets the student see their progress while reinforcing familiar concepts.

## Checkmark 

Each lesson ends with a summary check where the student answers a one or two short questions based on the day's activities.

## Worksheets

The 17 worksheets are designed to give the student a chance to think about and to practice the day's lesson. Many lessons have no worksheet. Worksheets are available in a separate book and are copywritten for a single student's use. Additional worksheet books may be ordered from RightStartMath.com.

# THE MATH GAMES

The games develop the players' math skills while they play. The players do not need to know their facts before playing. They will learn and practice their facts as they play. More importantly, the games give the players a reason to learn their facts.

Strategies provided in the daily lessons will give students confidence and independence. What is a simple step to someone who knows fractions often takes additional steps for a struggling learner. The variety of games and activities will support the process. Often a concept can be learned in more than one way, resulting in several games for the same concept.

Do not be in a hurry to get to the next lesson and game. Frequently go back to games already learned; the student will often play them from a new perspective. Game Day lessons will provide this review, although additional game play is strongly encouraged. Ideally, math card games will be played in addition to the lesson time.

## Description of the Cards

To play the daily games, you need two decks of special cards, which are available from RightStartMath.com. The descriptions are as follows:

### Fraction Cards

There are 75 fraction cards with 20 different fractions and 20 matching percent cards:

1. Two each of $\frac{4}{5}$, $\frac{7}{10}$, $\frac{9}{10}$.
2. Three each of $\frac{3}{4}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{5}{6}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{3}{10}$.
3. Four each of $\frac{2}{3}$, $\frac{1}{8}$, $\frac{1}{10}$.
4. Five each of 1, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$.
5. Eight each of $\frac{1}{2}$.
6. Twenty percent cards, corresponding to each fraction.

### Basic Number Cards

These 132 cards are numbered from 0 to 10. There are 12 of each number.

### Multiplication Cards

Each card corresponds to a number in the multiplication table from 1 × 1 to 10 × 10. Thus, it has 100 cards. Some numbers, such as 1, are found only once and others, such as 6, are repeated as often as four times.

## Where to play

For many players, the preferred place to play the games is on the floor. Children are more comfortable on the floor and the games seem more informal. A special rug used only for games makes a good playing area.

We have found that the Corners games are better suited for a table. This keep the smaller cards undisturbed by passing children and pets.

## The player with learning challenges

Often, those with learning challenges find memorizing unrelated facts very difficult and paperwork tedious. These games eliminate both problems and give the student a new approach to practicing their facts. Work in a place free from overwhelming noise and visual distractions. Repeat the games many times. The best way to end a game is saying, “Let’s play it again.”

# DR. COTTER'S THOUGHTS ON TEACHING MATH

## Problem Solving

Math is not about memorization, but understanding. Solving math problems is about thinking, not trying to recall a specific procedure.

Some people are under the impression there is only one way to solve any problem. Actually, solving a problem in different ways is a check for correctness, an important consideration in real life. Or another way to look at it: if humans didn't find new ways to solve problems, we would still be living in the Stone Age.

Generally, math problems need to be read several times. I told a group of middle-schoolers that even mathematicians read problems more than once. They were astonished.

Often a simple sketch can make a problem seem clearer. Some textbooks unnecessarily provide the picture. Students learn more by making their own sketch.

When a child gets really stuck, tell them to leave the problem and go do something else. Their brain will continue working in the background. When they return, they frequently have new insights.

Who has ever completed a puzzle by always finding the wanted piece on the first try? What baby has learned to walk without frustrations and falls? Studying math, or anything else for that matter, will be frustrating at times. Persistence is required for success. Sometimes math is thought of as exclusively a paper and pencil activity. On the contrary, what we write on paper is a shortcut for expressing a concept often found in some form in the real world.

## Fractions

Fractions have a bad reputation in the world of arithmetic as being incomprehensible and unpredictable. Cartoons exploit this fear of fractions for amusement. Believe or not, it's all a huge misunderstanding. Fractions are necessary and amazing. They are needed, among other things, for telling time, for counting money, and for cooking.

Historically, fractions were considered only as a part of a whole. For example, one third meant one of three equal parts of an object. Fractions could never be equal to or greater than one. The word fraction comes from Latin "frangere" meaning "to break."

Then in the 1600s, mathematicians expanded the fraction concept. Its new interpretation is the division of any two whole numbers. Thus, one third can be thought of as 1 divided by 3. This means $\frac{3}{3}$ is a now a fraction because it is 3 divided by 3, even though it is equal to one. And $\frac{4}{3}$ is a legitimate fraction because it is 4 divided by 3, even though the results are greater than one.

Some people didn't exactly welcome these new fractions. They called their old familiar fractions *proper* and the new unfamiliar fractions *improper*. The term improper seems to imply that something is not quite right, but it merely means improper fractions were not part of the traditional or original set of fractions. Most of the time, nobody cares if a fraction is proper or not.

In everyday language, unfortunately, we still use the centuries-old idea of a fraction. A small amount of something is referred to as a fraction of it. Even the dictionary says a fraction is a tiny part. The Thesaurus lists words such as "fragment" or "snippet" as a synonym for fraction. Thinking of fractions in this limited fashion is detrimental for a thorough understanding.

**Circular Fraction Model**

One of the earliest models used for teaching fractions was the circle. With the popularity of a certain Italian dish, we now talk about pizzas. However, pizzas or circles do have some serious drawbacks. While it's easy to see $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$ of a circle, it's not easy to see $\frac{1}{7}$, $\frac{1}{8}$, or $\frac{1}{9}$ or to visualize the differences. Trying to draw or compare fractions with pies is difficult because we tend to view quantities in a straight line, not around a ring.

**Linear Fraction Model**

A better visualizable fraction model is fraction strips. They are a series of ten rectangles with 1 written in the center of the first strip. The second strip has a vertical line dividing it in half with $\frac{1}{2}$ written in the centers of the two sections. The third strip has two vertical lines dividing the strip into thirds with $\frac{1}{3}$ written in all three sections. This pattern continues for the remaining strips to the tenths. The ten strips placed in order form the fraction chart.

You may have seen variations of these strips. Sometimes each fraction is a different color. At first, this might seem to be a nice enhancement. However, when the strips are assembled into the fraction chart, the colors predominate over the size of the fractions. Fractions must be identified by their differences in size, not by their color. All the strips must be the same color.

Another strange variation is the absent fractions. The sevenths and ninths go missing, but the twelfths are tossed in. Since when do we dismiss numbers we don't like? When the fraction chart is complete, there are some interesting arcs, actually hyperbolas. When some strips are missing, the hyperbolas disappear.

It is best to have two sets of fraction strips, one kept intact as the fraction chart and the other set cut into individual pieces. All children benefit by assembling the strips into the chart like a puzzle. Have them build the fraction chart next to, not on top of the model. Another intriguing activity is to take one strip of each size and build the stairs. Again, a hyperbola appears.

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# DAY 2 - Naming Fractions

This lesson will help the child begin to recognize the various fractions by size and in written format. Children need time to adjust to the concept that the larger the denominator, the smaller the fraction.

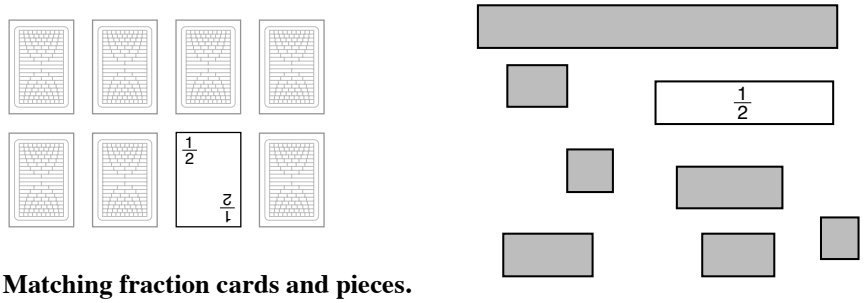
## "I will learn to name and compare fractions."

***You will need.*** The fraction chart, individual fraction pieces, and these eight fraction cards: 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$

***Assemble the fraction chart.*** Ask the child to assemble the fraction chart. If needed, use the completed chart for reference. Be certain to assemble the pieces next to the chart rather than on top of the completed chart.

***Unit fraction practice.*** Ask the child to find and to set aside the following fraction strips as you say them: 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$. Do not use the $\frac{1}{7}$ and $\frac{1}{9}$ at this time as the differences are visually minimal to the $\frac{1}{8}$ and $\frac{1}{10}$.

Lay the corresponding fraction cards face down on the table. Nearby lay out the eight fraction pieces face down. First, turn over a card. Then turn over a fraction piece. If they match, they collect both pieces. If they do not match, both the card and fraction piece are returned face down.




**Matching fraction cards and pieces.**

Continue until all the cards are collected. Practice until the child is able to match the cards and the fraction strips with ease.

***Finding fraction pieces.*** Ask the child to assemble the fraction chart. Then ask them to find and to set aside the following fraction strips as you say them:

| | | | |
|---|---|---|---|
| three $\frac{1}{4}$s | five $\frac{1}{8}$s | two $\frac{1}{9}$s | 4 sevenths |
| 5 ninths | 2 fifths | 5 tenths | 2 tenths |

Continue by writing, instead of saying, the fractions, for the child to find and set aside the corresponding fractions strips.

$\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{3}$ $\frac{1}{6}$

Also ask the child to participate in the fraction naming for the remaining strips. [the whole or 1, 1 half, 2 thirds, 2 fifths, 5 sixths, 3 sevenths, 2 eighths, 2 ninths, and 3 tenths]

# Unit Fraction War

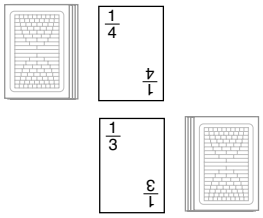
***You will need:*** The fraction pieces or the fraction chart. Generally the younger children prefer the pieces while the older children like the chart.

***Cards:*** Use all of the following unit fraction cards: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$, and five 1s.

***Number of players:*** Two or two teams.

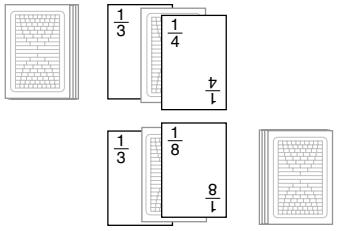
***Layout:*** Keep the cards face down and divide them evenly between the two players.

***Play:*** Each player takes the top card from his stack and lays it down in the middle of the table face up. The player whose card is greater takes both cards. Players alternate deciding whose card is greater.



**Unit Fraction War game in progress.**

Players continue comparing cards until they play identical cards, causing a "war." To resolve a war, both players play one card face down and then play a third card face up to be compared. The player who has the higher card in the last comparison takes all six cards.



**A "war".**

**Ask: Which is more, one half or one fourth? [one half] Which is more, one third or one? [one]**

# DAY 6 – Making One Half

Finding one half of something is a common task. Today's lesson and game will make it fun and help the child discover more ways to find one half.

## "I will learn which fractions combine to make one half."

***You will need.*** The fraction chart pieces, two identical containers, and water, rice, or dry beans

***Fractions equaling one half.*** Ask the child to assemble the fraction chart.

| 1 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\frac{1}{2}$ | | | | | $\frac{1}{2}$ | | | | |
| $\frac{1}{3}$ | | | $\frac{1}{3}$ | | | $\frac{1}{3}$ | | | |
| $\frac{1}{4}$ | | | $\frac{1}{4}$ | | | $\frac{1}{4}$ | | | $\frac{1}{4}$ |
| $\frac{1}{5}$ | | | $\frac{1}{5}$ | | | $\frac{1}{5}$ | | | $\frac{1}{5}$ |
| $\frac{1}{6}$ | | | $\frac{1}{6}$ | | | $\frac{1}{6}$ | | | $\frac{1}{6}$ |
| $\frac{1}{7}$ | | | $\frac{1}{7}$ | | | $\frac{1}{7}$ | | | $\frac{1}{7}$ |
| $\frac{1}{8}$ | | | $\frac{1}{8}$ | | | $\frac{1}{8}$ | | | $\frac{1}{8}$ |
| $\frac{1}{9}$ | | | $\frac{1}{9}$ | | | $\frac{1}{9}$ | | | $\frac{1}{9}$ |
| $\frac{1}{10}$ | | | $\frac{1}{10}$ | | | $\frac{1}{10}$ | | | $\frac{1}{10}$ |

**Complete fraction chart.**

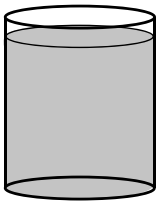
Ask them to take a one half piece and to set it aside. Ask: How many fourths does it take to equal one half? [2] Ask them to set them under the one half. Then ask: How many sixths does it take to equal one half? [3] Repeat for eighths [4] and tenths. [5] See the figure below.

| $\frac{1}{2}$ | | | | |
|---|---|---|---|---|
| $\frac{1}{4}$ | | $\frac{1}{4}$ | | |
| $\frac{1}{6}$ | | $\frac{1}{6}$ | | $\frac{1}{6}$ |
| $\frac{1}{8}$ | | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |
| $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ |

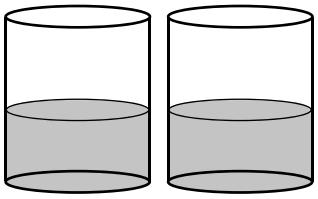
**Fractions equaling one half.**

Now ask the following questions: What is needed with $\frac{1}{4}$ to make $\frac{1}{2}$? [another $\frac{1}{4}$ or $\frac{2}{8}$] What is needed to make $\frac{1}{2}$ if there is $\frac{2}{8}$? [$\frac{1}{4}$ or $\frac{2}{8}$] What is needed to make $\frac{1}{2}$ if there is $\frac{4}{8}$? [nothing because $\frac{4}{8}$ equals $\frac{1}{2}$] Ask the child to name some other ways to make $\frac{1}{2}$.

***Finding half of the water.*** Set out two identical containers and fill one of them with water. Ask the child to pour half of the water from one container to the other container. Pouring back and forth from one container to the other may be necessary. Ask them to stop when half of the water is in each container. Ask: Which container has more water? [neither] Where is the whole? [the amount in both containers] Where are the parts? [each container] Point to one of the containers and ask: How much of the whole amount is in this container? [half] Repeat for the other container. [half] Ask: What do 2 halves make? [whole]



**The whole amount of water.**



**One half of the water poured into two containers.**

# One Half

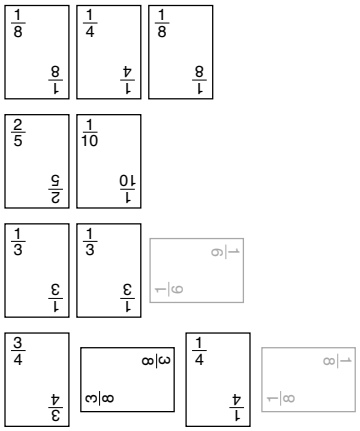
***Background:*** This game is designed to help the players become more sophisticated in adding and subtracting fractions informally, using the fraction chart as a whole and as pieces to explore and calculate.

***Cards:*** Sixty-seven fraction cards; all but the $\frac{1}{2}$ cards.

***Number of players:*** Two to four.

***Objective:*** To collect the most cards by completing a row.

***Deal:*** Each player is dealt four cards; the remaining cards form the stock. Following each turn, a player draws another card from the stock, keeping four cards in their hand.



**The goal is to make a row equal $\frac{1}{2}$. Indicate subtraction by turning a card sideways.**

***Play:*** The first player starts a row by placing a card on the table face up. Then the player to his left adds a card to that row or starts another row. Players take turns; the player laying the card that makes the row total $\frac{1}{2}$ collects all the cards in that row.

At first, the player will use simple fraction combinations as shown in the first two rows of the diagram. Later, encourage them to use more sophisticated combinations.

The cards in a row may overshoot the sum of $\frac{1}{2}$, which is then decreased by subtraction, indicated by turning the cards sideways.

It is not necessary or always desirable to add the cards in the order played. In the fourth row of the figure, the simplest approach is to add the fourths and eighths separately then subtract.

The game is over when all possible cards have been played. Some rows will not be completed at the end of the game.

**Ask: If you have $\frac{1}{4}$ of a glass of water, how many $\frac{1}{8}$s would you need to add to get $\frac{1}{2}$? [2]**

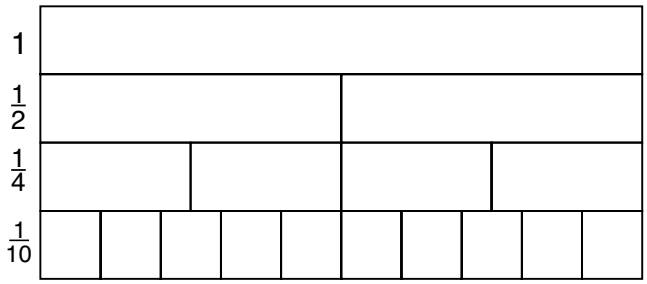
# DAY 8 – Fractions of a Dollar

We will review the parts of a dollar and relate the value of coins to fractions.

## "I will learn to relate the value of coins to fractions."

***You will need.*** Paper and pencil or dry erase board and marker, 50¢ (if available), 25¢, 10¢, 5¢ and 1¢ coins

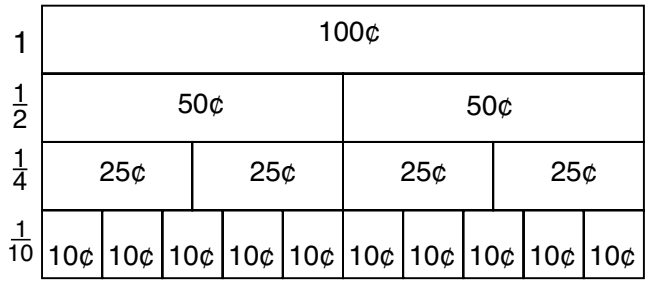
***Parts of a dollar.*** Draw a fraction chart with 1, halves, fourths, and tenths. Write the fractions at the left of the chart as shown below.




**Fractions strips.**

Ask: How much is half of a dollar, or half of 100? [50¢] Show them a half dollar, if available. Ask: What is it called? [half dollar] Explain half dollars are hardly ever used any more. Ask them to write 50¢ in the two rectangles, as shown.

Ask: How many cents are in a dollar? [100] Tell the child that the word *cent* means hundred. Ask: Can you think of any other words that start like cent and means hundred? [Century and centennial refer to one hundred years.] Write 100¢ in the top rectangle as shown below.




**Relating fractions to a dollar.**

Ask: How much is one fourth of a dollar, or half of 50¢? [25¢] Show them a quarter and ask: What is it this called? [quarter] Tell them that $\frac{1}{4}$ is sometimes called a quarter. Ask them to write 25¢ in the rectangles as shown above.

Ask: How much is one tenth of a dollar? This is a more difficult question; it might help to rephrase it. Ask: If 100 is broken into 10 parts, how much is in each part? [10¢] Show them the dime and ask: What is its name? [dime] Write 10¢ in the last row of rectangles.

Ask: What do you think the name is for the fraction that is half of one tenth? [one twentieth] What coin is worth half of a dime? [nickel] How much it is worth? [5¢] Show them the nickel and tell them it is larger than the dime because the dime used to be made from silver, which is very expensive so a smaller size for the dime was chosen. Explain that it is hard to make fractions that small, so you will not write them.

Finally show a penny and ask: What is it worth? [1¢] How many cents are in a dollar? [100] What fraction of a dollar is it? [$\frac{1}{100}$]


**50¢** **25¢** **10¢** **5¢** **1¢**

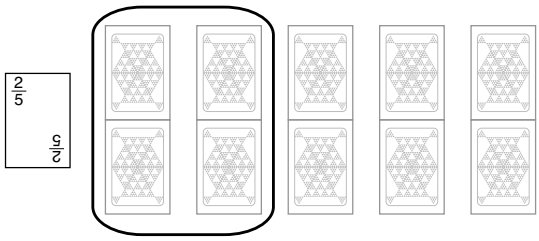
**The coins and their values.**

# Fraction of Ten

***Background:*** Fraction of Ten is played very much like What Fraction of Six, from the prior day's game.

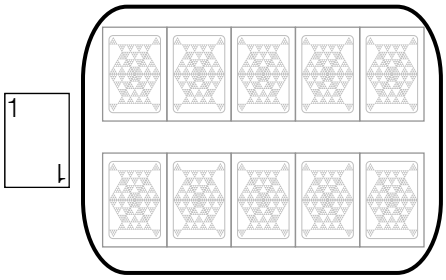
This time lay out 10 basic number cards face down and ask the player to take $\frac{1}{2}$ of them. First they must divide the cards into two groups and then take one of the groups. Show groups by pushing the cards together so their edges touch.

Now try $\frac{2}{5}$ of 10. Put the cards into groups of 5 see first figure at right. Then they takes two groups.




**$\frac{2}{5}$ of 10.**

Note that to take "1" of a group means to take the whole group. It is interesting to think of 1 as $\frac{2}{2}$. Divide the groups into 2 parts and take both of them.




**Taking the "1" or the "whole".**

***Cards:*** Use the fraction cards with 1s, halves, fifths, and tenths. Again, the basic number cards are used face down as counters.

***Number of players:*** Two or three players.

***Deal:*** Lay out 10 basic number cards face down, which are replenished after each turn.

***Play:*** The first player picks up a fraction card from the stock. He then collects that fraction of the 10 basic number cards.

After the basic number cards are replaced, the next player takes her turn. Turns continue until either stock is gone.

Just as in What Fraction of Six, the player to collect the most basic number cards wins the game.

**Ask: How many cents in a whole dollar? [100] How many cents in $\frac{1}{2}$ of a dollar? [50] How many cents in $\frac{1}{4}$ of a dollar? [25] How many cents are in $\frac{1}{10}$ of a dollar? [10] How many cents are in $\frac{1}{100}$ of a dollar? [1]**

# DAY 14 – More Equivalent Fractions

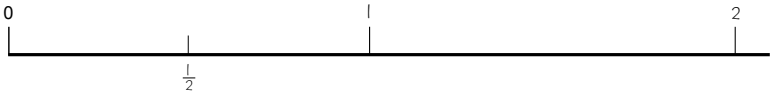
Sometimes a new approach will increase and solidify concepts. Today we will explore equivalent fractions with paper and pencil.

## "I will work with equivalent fractions."

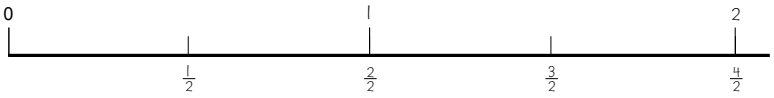
***You will need.*** Worksheet 5, More Equivalent Fractions

***Worksheet 5.*** On the worksheet, using the top thick horizontal line, tell the child to mark and label the 1 (leaving room for 2) as shown.

Ask: Where would you put a mark for 2? Have the child make the mark and write the number. Ask: Where would you put the mark for $\frac{1}{2}$? Tell them to make the mark on the thick line and write $\frac{1}{2}$ on the first line below, as shown.

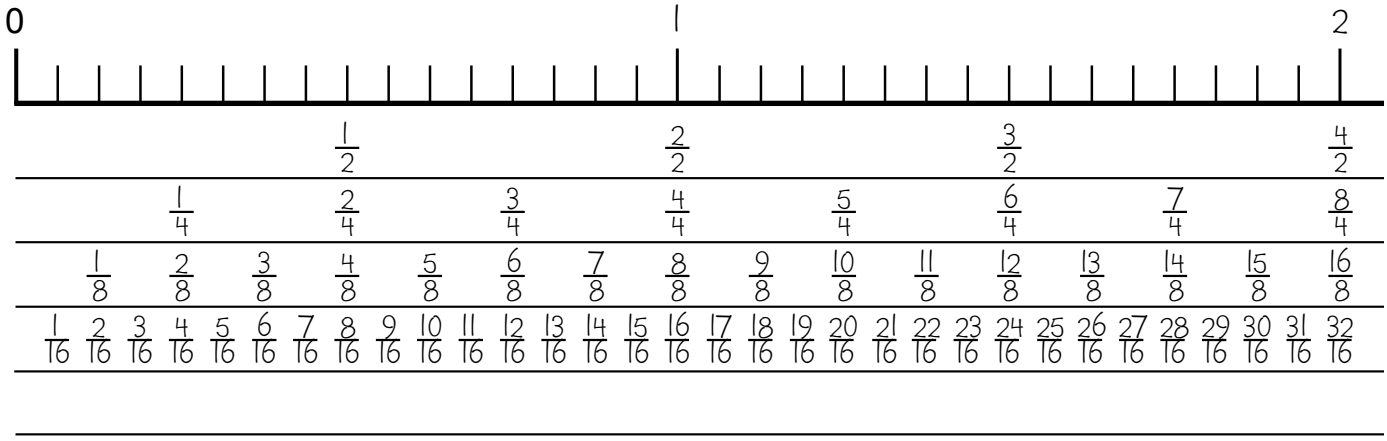



Ask: How many halves is 1? [$\frac{2}{2}$] Tell them to write $\frac{2}{2}$ below the 1, as shown below. Ask: Where would you put the mark for $\frac{3}{2}$? Tell them to mark and write $\frac{3}{2}$. Ask: How many halves is 2? [$\frac{4}{2}$] Have the child write that below the 2.




***Adding to the fraction number line.*** Use the fraction chart for reference when necessary. Ask: Where would you put the mark for $\frac{1}{4}$? Tell the child to make the mark and write $\frac{1}{4}$ on the second line, as shown below. Ask: How many fourths is in $\frac{1}{2}$? [$\frac{2}{4}$] Tell them to write $\frac{2}{4}$ below the $\frac{1}{2}$ as shown. Have the child mark and write $\frac{3}{4}$. Pointing to the 1 and $\frac{2}{2}$, ask: What would we write here? [$\frac{4}{4}$] Tell the child to continue marking and writing $\frac{5}{4}$, $\frac{6}{4}$, $\frac{7}{4}$, and $\frac{8}{4}$ as shown.

Ask: Where would you put the mark for $\frac{1}{8}$? Tell the child to make the mark and write $\frac{1}{8}$ on the third line. Continue by labeling and marking the remaining eighths as shown below. Continue with the $\frac{1}{16}$s, also shown. If the child is interested, complete the chart with the $\frac{1}{32}$s, not shown.



# Follow the Series

***Background:*** This game is designed to provide more practice with fraction series. The objective is to collect the most cards by playing the highest card of the series lead.

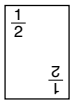
***You will need:*** Fraction charts for reference.

***Cards:*** Use the 75 fraction cards for three or five players. Remove any three cards for four or six players.

***Number of players:*** From three to six.

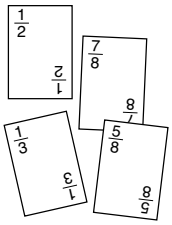
***Deal:*** Deal all the cards to the players. After the deal, each player picks up their top four cards. After each round, the players pick up another card keeping four cards in hand.

***Play:*** The first player to the left of the dealer lays down a card and announces, if necessary, which series he wants. For example, if he lays down a $\frac{1}{2}$, he may call halves, fourths, or eighths.



**Player One starts by laying $\frac{1}{2}$ and calling "eighths".**

The following players, in turn, each lay down a card from that series if they have one. If they do not have one, they may play any card. The person playing the highest card in the series takes all the cards on the table and plays a new card and calls the series for the next round. The 1s belong to every series. In the event that two players lay down the same high card, the first one to play it takes the cards.



**The player who laid down the $\frac{7}{8}$ takes all the cards of this round and starts the next round.**

**Ask: Which is bigger $1\frac{1}{4}$ or $\frac{5}{4}$? [they are the same]**

# DAY 29 – Fraction Addition

People add fractions in everyday life. Consider measuring $\frac{3}{8}$ inches and needing to add $\frac{1}{4}$ inches.

## "I will add fractions with and without a common denominator."

***You will need.*** Fraction chart and pieces, paper and pencil or dry erase board and marker, and a multiplication table

***Adding fractions with a common denominator.*** Ask the child to write the fractions $\frac{2}{5}$ and $\frac{1}{5}$ in a horizontal addition equation as shown at left. **$\frac{2}{5} + \frac{1}{5} =$**

When the denominators are the same, adding the fractions is very simple. Ask them to find $\frac{2}{5}$ on the fraction chart. Then have them add another $\frac{1}{5}$. Ask: What is total amount? [$\frac{3}{5}$] Tell them to complete the written equation by adding the numerators, keeping the denominator the same, as shown. **$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$**

Have the child practice with $\frac{3}{8} + \frac{2}{8}$, [$\frac{5}{8}$] $\frac{3}{10} + \frac{3}{10}$, [$\frac{6}{10}$] and $\frac{5}{7} + \frac{1}{7}$. [$\frac{6}{7}$]

***Adding unit fractions without a common denominator.*** When the denominators are not the same, we must first change the fractions into equivalent fractions with common denominators. We will then be able to add the numerators. The common denominator that is typically used is the lowest common multiple or LCM.

Ask the child to write the fractions $\frac{1}{2}$ and $\frac{1}{3}$ as shown at left. **$\frac{1}{2} + \frac{1}{3} =$**

Have the child find $\frac{1}{2}$ on the fraction chart and then ask them what fraction they think they would have, if they added $\frac{1}{3}$ to the $\frac{1}{2}$. Let the child use the fraction chart and pieces to experiment. [$\frac{5}{6}$]

Pointing to the written equation, say: Since the denominators are not the same, the first step we must take is to find the LCM of 2 and 3. [6] Then convert the fractions into their equivalents, so $\frac{1}{2}$ becomes $\frac{3}{6}$ and $\frac{1}{3}$ becomes $\frac{2}{6}$. **$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$**

Have the child write the equation using the equivalent fractions. Point to the equivalent fractions on the fraction chart and then have them complete the equation. [$\frac{5}{6}$]

Repeat for $\frac{1}{2} + \frac{1}{5}$ [$\frac{5}{10} + \frac{2}{10} = \frac{7}{10}$] and $\frac{1}{4} + \frac{1}{3}$. [$\frac{3}{12} + \frac{4}{12} = \frac{7}{12}$]

***Adding non-unit fractions.*** Ask the child to write the equation $\frac{2}{5} + \frac{3}{7}$. If needed, remind the child that when the denominators are not the same, they must first change the fractions into equivalent fractions with common denominators.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

Ask: What is the common denominator? [35] Find $\frac{2}{5}$ on the multiplication table. Tell them to place one finger on the 2 and the other finger on the 5 then slide their fingers from the $\frac{2}{5}$ as a pair to the right until they find the common denominator of 35 as shown. Ask: What is $\frac{2}{5}$ equivalent to? [$\frac{14}{35}$] How are the denominators related; the 5 in $\frac{2}{5}$ and the 35 in $\frac{14}{35}$? [7 times larger, 5 × 7 = 35] How are the numerators related; the 2 in $\frac{2}{5}$ and the 14 in $\frac{14}{35}$? [7 times larger, 2 × 7 = 14]

Now find the next fraction, $\frac{3}{7}$, on the multiplication table. Place one finger on the 3 and the other finger on the 7, sliding the fingers as a pair from the $\frac{3}{7}$ until they find the common denominator of 35.

Ask: What is $\frac{3}{7}$ equivalent to? [$\frac{15}{35}$] How are the denominators related? [5 times larger, 7 × 5 = 35] How are the numerators related? [5 times larger, 3 × 5 = 15] So what is $\frac{2}{5} + \frac{3}{7}$ (or $\frac{14}{35} + \frac{15}{35}$)? [$\frac{29}{35}$] Repeat for $\frac{1}{4} + \frac{3}{5}$ [$\frac{5}{20} + \frac{12}{20} = \frac{17}{20}$] and $\frac{4}{7} + \frac{3}{8}$. [$\frac{32}{56} + \frac{21}{56} = \frac{53}{56}$]


# Fraction Addition

Most of the fractions added in this game will require finding a common denominator by finding the LCM.

***You will need:*** Paper and pencil or dry erase board and marker and a multiplication table

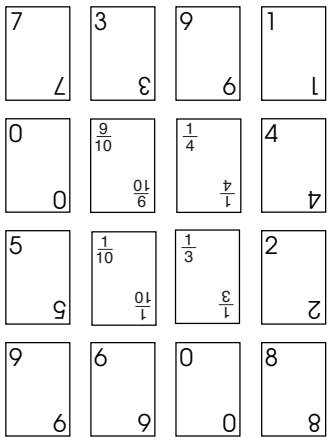
***Cards:*** Use all the fraction cards and basic number cards, except the 10s.

***Number of players:*** Two to four. Partners can also play.

***Objective:*** To add the fraction cards either horizontally or vertically and find the sum with the basic number cards.

***Deal:*** Arrange four fraction cards in two rows with two each. Then place 12 basic number cards around them as shown. If three or more basic number cards are the same, replace them.

Deal three basic number cards to each player. After a turn, a player picks up enough basic number cards to keep three cards in hand.




**Layout for Fraction Addition.**

***Play:*** All the players add the two rows and columns of fractions. The first player looks for the sums in the basic number cards surrounding the fraction cards and in their hand. Two groups of fractions may be collected during a turn. Replace any collected cards from the respective stocks after each turn.

If no one can play, replace the four basic number cards in the corners. Players take turns until the basic number cards are exhausted.

In the layout shown, the cards $\frac{9}{10}$ and $\frac{1}{10}$ could be collected since their sum is 1. The remaining sum of $\frac{1}{4}$ and $\frac{1}{3}$ is $\frac{7}{12}$.
The 7 and 2 cards are available, but the player needs an additional 1 in his hand to claim the cards.

Alternatively, the player could collect cards 1, 3, 2, and 0 because the sum of $\frac{9}{10}$ and $\frac{1}{4}$ is $\frac{23}{20}$. Because the remaining sum of $\frac{1}{10} + \frac{1}{3}$ is $\frac{13}{30}$, the player could pick up the cards only if she has a 1 and two 3s in her hand. The fractions must be simplified before collecting cards.

The person or partnership collecting the most basic number cards is the winner.

***Variation:*** This game can also be played as fraction subtraction which will be taught in the next day's lesson.

**Ask: What is the first step when adding fractions with different denominators? [finding the common denominator]**

# DAY 33 – Fraction Multiplication

It is a useful tool to be able to visualize multiplying a fraction by a fraction. The fraction pieces will help us see what it looks like.

**"I will multiply fractions by a fraction and fractions by a whole number."**

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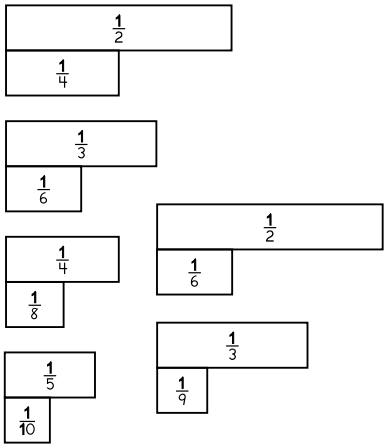
***You will need.*** Fraction pieces, 12 cards for counters, and paper and pencil or dry erase board and marker

***Fraction of a fraction.*** Give the child the fraction pieces and tell them to find a $\frac{1}{2}$ fraction piece. Ask: What fraction piece is half of this amount? Tell the child to find the fraction piece; allow them to explore using trial and error if necessary. [$\frac{1}{4}$] It may help to turn the pieces over so that written numbers are not distracting. Set both pieces aside. See figure.




Tell the child to find a $\frac{1}{3}$ fraction piece. Ask: What fraction piece is half of this amount? Again, let the child explore. [$\frac{1}{6}$] Do not discuss relationships and algorithms at this point. Set both pieces below the first two pieces as shown on the right. Continue finding half of $\frac{1}{4}$ [$\frac{1}{8}$] and $\frac{1}{5}$ [$\frac{1}{10}$] and setting the pairs aside.

Now tell the child to find the second $\frac{1}{2}$ fraction piece. Ask: What fraction piece is a third of this amount? [$\frac{1}{6}$] Again, let the child explore. If assistance is needed, turn the fraction piece over so the numbers are not a distraction and say: What three equal fraction pieces are needed to make this amount? [$\frac{1}{6}$s] Set the pair in a new column as shown on the far right. Tell the child to find a third of $\frac{1}{3}$ and set it aside. [$\frac{1}{9}$] Keep all the pairs on the table.

**Of *means multiply.*** To present the meaning of the word *of* in multiplication, tell the child to lay out four cards face down in a row. Ask: How much is 1 group of 4? [4] How much is 2 groups of 4? [8] Lay a second set of four cards if necessary. Ask: How much is 3 groups of 4? [12] Point out that the word *of* means to multiply. Tell the child we write this as $4 \times 3$.

Point to the first fraction pair from above, $\frac{1}{2}$ and $\frac{1}{4}$, ask: What is one half of one half? [$\frac{1}{4}$] What is $\frac{1}{2}$ of $\frac{1}{3}$? [$\frac{1}{6}$] What is $\frac{1}{2}$ of $\frac{1}{4}$? [$\frac{1}{8}$] What is $\frac{1}{2}$ of $\frac{1}{5}$? [$\frac{1}{10}$] Point to the next group and ask: What is $\frac{1}{3}$ of $\frac{1}{2}$? [$\frac{1}{6}$] What is $\frac{1}{3}$ of $\frac{1}{3}$? [$\frac{1}{9}$] Remind the child that *of* means multiply and they just multiplied fractions!

***Fraction of a whole number.*** Lay out 12 cards in a row face down. Ask the child to find $\frac{1}{2}$ of the 12 cards. [6] They may move the cards around for grouping. Have them write the equation: $\frac{1}{2} \times 12 = 6$.

Now ask the child to find $\frac{1}{3}$ of 12. [4] If necessary, remind them that we are grouping 12 into three equal groups, which is four cards in each group, just like they grouped 12 into two equal groups of six cards each. Have them write the equation: $\frac{1}{3} \times 12 = 4$.

Next ask the child to find $\frac{2}{3}$ of 12. [8] The cards will need to be grouped in thirds, then two groups of four identified. Have them write the equation: $\frac{2}{3} \times 12 = 8$. Continue with $\frac{1}{6}$ of 12 [2; $\frac{1}{6} \times 12 = 2$], $\frac{5}{6}$ of 12, [10; $\frac{5}{6} \times 12 = 10$], and $\frac{3}{4}$ of 12. [9; $\frac{3}{4} \times 12 = 9$]

Some children will discover the algorithm and others will not yet. Don't point it out as this will be addressed in an upcoming lesson.

# Fraction of Twelve on Paper with Cards

***You will need:*** Each player will need paper and pencil or dry erase board and marker.

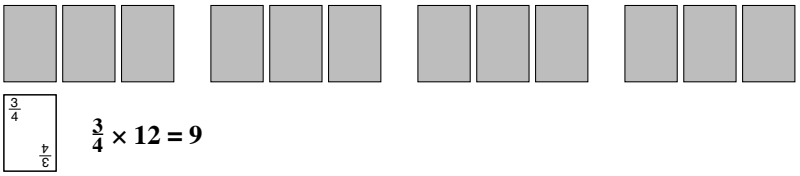
***Cards:*** Use the halves, thirds, fourths, and sixths from the fraction cards and 12 cards from any other deck as counters for each player.

***Number of players:*** From two to six.

***Deal:*** Distribute the fraction cards evenly among the players. Set aside any extra cards.

***Play:*** Each player turns over one of her fraction cards, writes it down, and follows it by "× 12." Then she figures out the product and writes it down. She continues with her remaining cards. To make the addition of the found products easier, all the equations should be written directly below each other, as shown.

$\frac{1}{3} \times 12 = 4$



$\frac{3}{4} \times 12 = 9$

Players work without regard to turn. After figuring the products for their cards, the players add them together. The winner is the player with the highest total.

# Fraction of Ten on Paper with Cards

***Background:*** This game is the same as the preceding game but played with ten cards.

***Cards:*** Use the 1s, halves, fifths, and tenths from the fraction cards and use ten cards from any other deck as counters for each player.

***Number of players:*** From two to six.

***Play:*** Same as Fraction of Twelve on Paper with Cards, but each card is multiplied by 10.

# Fraction of Sixteen on Paper with Cards

***Background:*** This game is played with 16 cards.

***Cards:*** The halves, fourths, and eighths from the fraction cards and any 16 cards from another deck for each player.

***Number of players:*** From two to six.

***Play:*** Same as Fraction of Twelve on Paper with Cards but each card is multiplied by 16.

**Ask: What is $\frac{1}{2}$ of 12? [6] What is $\frac{1}{2}$ of $\frac{1}{2}$? [$\frac{1}{4}$]**