RIGHTSTART™ TUTORING

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NUMBER SENSE

Activities for Learning, Inc.

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INTRODUCTION

Welcome! This manual is about the application and use of number sense. It is intended for those who have a weak, incomplete, or non-existent understanding of number sense. The most frequent reason for confusion with number sense is a vague understanding of what numbers mean and how they are related to each other.

Attempts have been made to solve this by focusing on rote memorization without much comprehension. For many, the burden of memorization is overwhelming, never mind the frequent need to review. Students who have memorization without understanding struggle to apply their skills to new situations. This results in frustration, confusion, and an aversion to math.

On the contrary, we now know that a deep understanding of concepts removes anxiety, lessens the burden of memorizing, makes advanced math easier to grasp, and makes math more enjoyable.

It does not matter if the student is 9 years old or 90 years old; these lessons will approach number sense with a new perspective that follows the RightStart Mathematics approach and philosophy.

So, what is number sense? It is the understanding of numbers and the quantities they represent. It is also the relationship between numbers and the operations that can affect numbers, such as addition and subtraction. Number sense provides order to math and provides a strong foundation needed for future math learning.

There are a number of things in this manual that will be different from the traditional way number sense is taught. Although they will be explained in greater detail during the appropriate lessons, here is a quick overview.

Counting versus Subitizing

From a very young age, children are taught to count before they begin their formal education. This counting process is the traditional approach for adding and subtracting, yet quickly becomes a problem, especially with larger quantities and multiplication.

Rather than relying on counting, we will have the student see quantities in groups of fives and tens. This allows for quantities to be quickly recognized, or subitized. It also allows the quantities to become visualizable, that is, to be seen mentally.

The primary tool used throughout this manual is the AL Abacus, which is grouped in fives and tens. Strategies for addition and subtraction will also incorporate and utilize grouping. With frequent and consistent use, the student will develop a mental image of the abacus and strategies, thereby removing the need for the physical manipulative.

An additional reason to use the abacus is that the area of the brain that controls the fingers is adjacent to the math area of the brain. The motion of the fingers while using the abacus will stimulate the surrounding areas of the brain. The same does not apply to using fingers for counting.

If a child struggles or reverts to counting, tell them to use their abacus. It will not become a crutch; rather, with repeated use, the child will develop a mental image of the abacus they can rely on.

Place Value

In many Asian languages, numbers are said as ten-1 for eleven, ten-2 for twelve, ten-3 for thirteen, and so on. The twenties are read as 2-ten 1, 2-ten 2, 2-ten 3, and the thirties are read as 3-ten 1, 3-ten 2, 3-ten 3, and so on up to 9-ten 9. This way of saying numbers makes place value readily understood, in other words, transparent. Therefore, transparent number naming, also called the math way of saying numbers, will temporarily be used in this curriculum.

Many European languages, including English, names for numbers from 11 to 99 are confusing. The words eleven, twelve, thirteen, and so on, do not help the child understand tens and ones, which is the foundation of place value. Many English speaking children do not realize that 13 is 10 and 3 more. Without understanding place value, it is more challenging to work with larger numbers.

Lessons will identify how the math way of saying the numbers connects to the English way of saying the numbers. This makes place value clear and easy to use. Understanding place value makes the strategies for addition and subtraction effective and powerful.

Older students will likely catch on to the pattern of the transparent number naming quickly. If they understand the two ways of saying the numbers, they can use both the traditional names and the transparent way of number naming during the lessons.

Math Card Games

Most students are overwhelmed with math worksheets. Students who are not understanding something will not benefit from more and more worksheets. Flash cards just reinforce what a student doesn't know. They can become another source of frustration and feelings of failure. Rather than worksheets or flashcards, games will be used with this manual.

These math card games will allow the student to learn and practice new skills. Games keep math time enjoyable. Emotional states are stored along with what has been learned. If a student has an enjoyable time learning, then positive emotions will replace past negative emotions.

A game will be assigned in each lesson. Some games are solitaire games and some are for two or more players. Include other family members in the games. There is nothing more powerful than a child playing a game against their parent—and winning!

Instructions are given for each game. Adapt as necessary to fit the child and the situation. For example, turn the games into one-person games or modify to fit more than one player. Please contact RightStart Math if you need ideas for modifying the games.

It is impossible to overplay the games. The games will hone skills and help the student become more confident and fluid in their thinking. The more games are played, the more the student learns. If a concept is not solid, play the game again. Also, playing past games will allow the student to enjoy their growth and master their facts.

Multiple Approaches

Multiple approaches will be presented to solve addition and subtraction problems. These are not given to confuse a student, rather, they provide options. One strategy might become the student's favorite, but the next day's strategy could be even better. Multiple approaches give the student additional perspectives to expand their understanding.

If a strategy or approach does not resonate with you as the teacher, that does not mean it won't be important to the student. Follow the lessons, even if you are unsure yourself, because it may be critical to the student.

For example, the subtraction algorithm, or procedure, generally taught in the United States is not the only one is general use; students in Latin America use another algorithm. In fact, there are at least seven methods.

In a simpler method, the work proceeds from left to right like division, rather than right to left like addition. According to research, it is easier for most children to complete the work for trading, or borrowing, before performing the actual subtracting.

Summary

The lessons, activities, and games in this program are from the RightStart™ Mathematics curriculum and from *Math Card Games, 5th edition*, both written by Dr. Joan A. Cotter. This manual can be used alongside any math program; knowledge of the RightStart™ Mathematics program is not required.

This manual will provide the teaching guide and will make learning interesting with games and activities. If a student struggles, slow down the lesson and concentrate on the activities and games. Make sure they are using the AL Abacus. Refer to numbers using the transparent number names.

In these 50 days of lessons, a solid foundation of number sense will be laid while proceeding step by step to develop clear understanding. There are no worksheets, rather daily games will provide practice and review.

We believe that through these lessons and games, students will develop a renewed interest and enjoyment in mathematics, thereby enriching their lives. We also hope many of them will become tomorrow's mathematicians, scientists, and engineers.

We want you and your students to have great success in learning and discovering number sense. Let us know how this tutoring program benefits you and your students. Please share your experience and keep in touch!

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DAILY LESSONS

Needed Materials

Materials needed for the day's activities will be identified at the beginning of the lesson. Periodically, paper and pencil or a dry erase board and marker will also be needed. If an appendix page is needed, it will be listed.

The AL Abacus will enable the student to build a mental model necessary for concept formation. Even if a child knows a fact, say 5 + 5, it is important they also see it physically on the abacus. This helps with basic number sense, as well as develops understanding of the relationships between numbers and the operations that can modify them.

Manipulatives are not to be regarded as crutches, rather tools for learning. In practice, the student will refer to them less and less and finally not at all. Sometimes just the security of having them nearby helps, even if they are not used. Let the student decide when they no longer need them.

Activities

This section is the heart of each day's lesson. These are the instructions for teaching the lesson. The expected answers from the student are given in square brackets.

Research shows that the quiet time for thoughtful response should be about three to five seconds. Avoid talking during this quiet time; resist the temptation to rephrase the question. This time gives the slower student time to think and the quicker student time to think more deeply. Encourage the student to develop persistence and perseverance. Avoid giving hints or explanations too quickly. Students, and people in general, tend to stop thinking once they hear the answer.

Games

Daily games, not worksheets or flash cards, provide practice of the new skills. The games can be played as many times as necessary until proficiency takes place. They are as important to learning math as books are to reading. Reviewing old games lets the student see their progress while reinforcing familiar concepts.

Worksheets

There are no worksheets for this tutoring manual. Practice will come from the games.

There will be situations where equations may be written out. Paper and pencil or a dry erase board and marker will be needed. Some children may struggle with using paper and pencil, yet will find a dry erase board and marker smoother and easier to work with. Use the child's preferred medium. If you need or want to record work from a dry erase board, take a picture, then save it for your records.

There are some children who find the simple act of writing uncomfortable, painful, or just overwhelming. We recommend the teacher becomes the scribe, writing exactly what the student says, even if it's a wrong answer.

THE MATH GAMES

The games develop the players' math skills while they play. The players do not need to know their facts before playing. They will learn and practice their facts as they play. More importantly, the games give the players a reason to learn their facts.

Strategies provided in the daily lessons will give students confidence and independence. What is a simple step to someone who knows addition and subtraction often takes additional steps for a struggling learner. The variety of games and activities will support the process. Often a concept can be learned in more than one way, resulting in several games for the same concept.

Do not be in a hurry to get to the next lesson and game. Frequently go back to games already learned; the student will often play them from a new perspective. Game Day lessons will provide this review, although additional game play is strongly encouraged. Ideally, math card games will be played in addition to the lesson time.

Description of the Cards

To play the daily games, you need two decks of special cards, which are available from Activities for Learning, Inc. The descriptions are as follows:

Basic Number Cards

These 132 cards are numbered from 0 to 10. There are 12 of each number.

Corners™ Cards

The Corners™ cards each have four colored numbers between 1 and 10. There are 50 Corners cards and no two cards alike.

Where to play

For many players, the preferred place to play the games is on the floor. Children are more comfortable on the floor and the games seem more informal. A special rug used only for games makes a good playing area.

We have found that the Corners games are better suited for a table. This keep the smaller cards undisturbed by passing children and pets.

The player with learning challenges

Often, those with learning challenges find memorizing unrelated facts very difficult and paperwork tedious. These games eliminate both problems and give the student a new approach to practicing their facts. Work in a place free from overwhelming noise and visual distractions. Repeat the games many times. The best way to end a game is saying, "Let's play it again."

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DAY 1 – Subitizing

Needed Materials. 9 Basic Number cards (to be used face down), AL Abacus, Finger cards (Appendix p. 1) and Bead cards (Appendix p. 2) cut apart

> *NOTE: Subitizing (SOO bih tighz ing) is the ability to perceive a quantity at a glance without counting. When counting, the child focuses on one item at a time. Subitizing allows the child to simultaneously see the whole and the individual items. It is easier for children (and adults) to subitize quantities than to count them. To foster natural subitizing skills, children should be **discouraged from counting** small quantities.*
>
> *Researcher Dr. Karen Wynn discovered that five-month-old babies can subitize up to three objects and many 12-month-old babies can subitize up to four objects. Researchers also show that children who can represent quantities with their fingers, not by counting but by subitizing, score better in upper elementary math.*

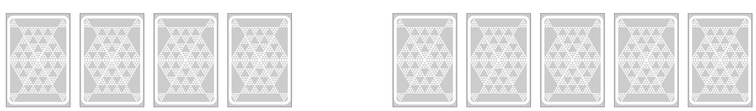
Quantities 1 through 5. Tell the child to show 2 with the fingers on their left hand. When facing the child, use your right hand to show 2 so the child can mirror you. If you are sitting next to the child, use your left hand. Repeat for 3, 1, 5, and 4.

> *NOTE: Using fingers to show quantity is a common action. The left hand is used for showing numbers five and less which correlates with reading from left to right. It does not matter which fingers of the left hand are used.*
>
> *If the child is confident and quick to display the quantities, show the quantities on your hand slower than the child. If the child is unsure of the quantities, quickly show the quantities on your hand so they can correctly display the quantities. Once they become more confident, slow your timing and allow them to display the quantities before you.*

Explain to the child that naming quantities without counting is called *subitizing*. Say the numbers again in random order while the child shows the amount with their fingers. Then raise various fingers on your hand showing quantities 1 through 5 and ask them to name the amount.

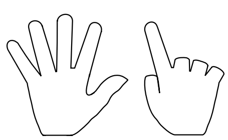
Comparing 4 with 5. Lay 4 cards face down and another group of 5 face down. See the figure below. Ask: Which group has a middle card, 4 or 5? [5] Say: This will help you subitize 4 and 5.

If the middle card is not obvious to the child, tell them to point to the first card with their left hand and the last card with their right hand. Tell them to simultaneously point to the second and fourth cards. Five has a middle card, whereas four does not.

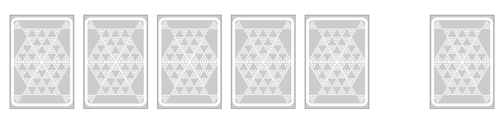


Comparing four and five: five has a middle card.

Quantities 6, 7, and 10. Tell the child to show 6 with their fingers as shown below on the left. Be sure the child uses their left hand for five and their right hand for amounts over five. If needed, demonstrate the quantities on your hands so the child can mirror you until they become confident.



Six.



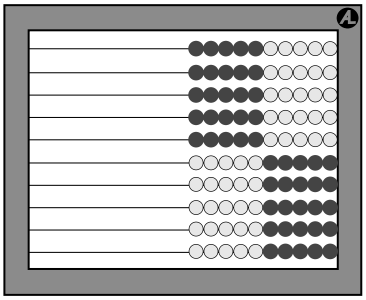
Then ask them to make 6 with the cards. Tell them to leave a gap between the five and the one, just like their hands.

Repeat the process for 7 using their fingers and the cards. Do the same with 10. Ask: What makes 10 a special number? [all the fingers, two groups of five]

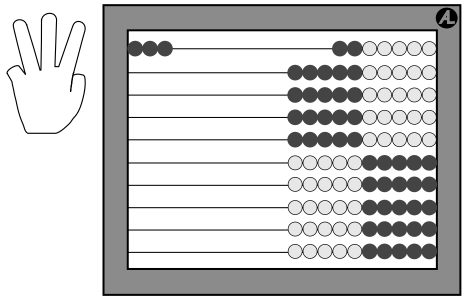
> *NOTE: The quantity of 8 is the hardest quantity to subitize. Quantities 8 and 9 will be addressed in the next lesson.*

Introducing the AL Abacus. Give the child the abacus. Ask: Are the two sides of the abacus the same? [no] Say: On one side each bead has a value of one. On the other side, a bead's value depends upon which column it is in.

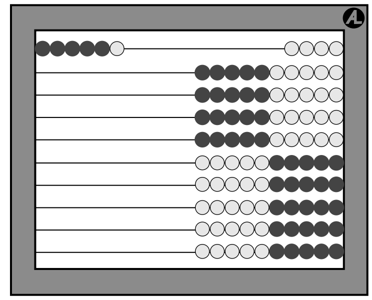
Tell the child to place their abacus flat with the wires horizontal and the logo on the top right. Tell them this is the front of the abacus. See the first figure below. Tell them to clear the abacus by lifting the left edge so the beads slide to the right side toward the logo.



Abacus cleared.



Three on fingers and the abacus.



Six.

Ask the child to show 3 on their fingers, then enter 3 on the abacus by sliding 3 beads **as a group** on the top wire to the left edge. See the second figure above. If the child starts counting the beads, challenge them to move the beads without counting. If they are interested or need more practice, have them enter 3 on each wire.

NOTE: Some children may need a slight gap in the row of 10 beads to see the quantity of 3. If so, provide a small nudge between the third and fourth beads, then allow them to move the beads as a group. Provide a smaller and smaller gap until they can enter 3 without assistance and, more importantly, without counting.

If they enter six rows of 3, the color change from blue to yellow may create a small pause. Allow them to think it through without any comment from you. If needed, simply remind them they are entering 3.

Now ask them to clear the abacus and show 6 on their fingers, then enter 6 on the abacus without counting. Ask: How does the abacus compare to your hands? [Blue beads are like the left hand; yellow beads are like the right hand.] See the third figure above. Repeat for 7 and 10.

Then ask the child to enter numbers 1 through 7 and 10 in random order on the abacus. If they are able, encourage them to enter the new number without clearing the previous number. For example, if 4 is already entered and 7 is the next number, only 3 beads need to be added. Finally, randomly enter numbers on the abacus from 1 through 7 and 10 for them to read.

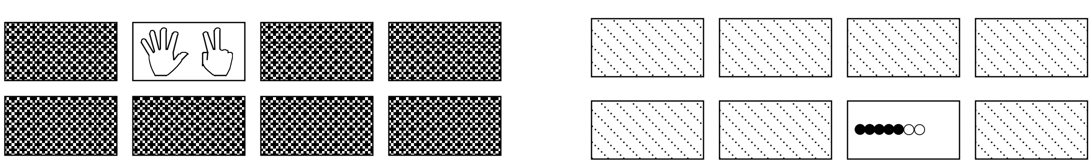
Game 1 Finger and Bead Card Memory

This memory game provides practice identifying and matching quantities. Use the finger cards and bead cards from the appendix pages in the back of this lesson book. Remove the 8 and 9 cards from each set and put aside.

Lay out the finger cards face down in two rows. Lay the bead cards face down to the right of the finger cards. See the figure below. The object of the game is to collect the most matching pairs.

The first player turns over a finger card, states the quantity, then turns over a bead card. If they match, he collects both. If not, he turns both cards face down. The next player takes her turn.

Players continue taking turns until all the cards are collected. No extra turns are taken when the correct cards are found.



Play the game a few times until the child is comfortable and confident with subitizing.

DAY 5 - Ones, Tens, and Hundreds the Math Way

Needed Materials. AL Abacus, Place-value cards 1 through 9, 10 through 90, and 100 through 900, paper and pencil or dry erase board and marker, Abacus Tiles (Appendix p. 3)

Naming quantities. Ask the child to enter 3-ten on their abacus and find the matching place-value card. Continue with 8-ten, 7, 9-ten, and 8. Then ask the child to enter 4-ten 6 on their abacus and compose the number with place-value cards. Repeat for other numbers, such as 6-ten 9, 2-ten 5, 7-ten 4, 10-ten, and more as needed.

Enter various quantities on the abacus and ask the child to name them using the math way of number naming. Enter 7-ten and ask: How much is it? [7-ten] Enter 6-ten 8 and ask: How much is it? [6-ten 8] Continue as needed, focusing on the tens greater than 5-ten.

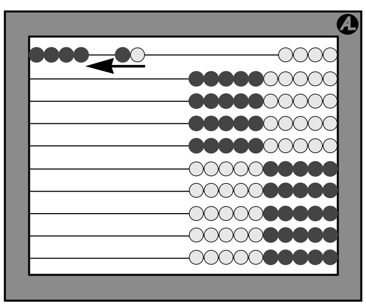
> *NOTE: It is acceptable if a child has a two to three second pause before providing the answer. This time allows them to see the groups of fives and tens.* ***Make sure they are not counting the quantities.*** *If they are unsure of a quantity, have them display it using their fingers, then point out the five and the additional amount. For example, if they are slow to name 8-ten (80), have them show 8 on their fingers; 5 on the left and 3 on the right. Point out the 5-ten beads starting with blue, then the 3-ten beads starting with yellow, grouping 8-ten just like 8 on their hands.*

Adding. Write 6 + 1 = ____ where the child can see it. Say: Enter 6 on your abacus. Add another bead. Ask: What is the sum, or the total? [7] Clear the abacus and write 7 + 2 = ____. Say: Enter 7 on your abacus. Add 2 more beads. Ask: What is the sum? [9] Clear the abacus again and write 3 + 4 = ____. Have the child enter the beads and find the sum. [7]

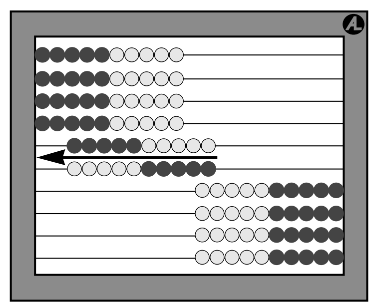
> *NOTE: Regarding 3 + 4, some children will pause with the quantity of four beads being in two different colors. Give them time to think. Avoid giving hints. If needed, have them show 4 on their fingers, then subitize 4 beads and enter.*

Continue simple equations with sums of 10 or less until the child is confident.

Adding tens. Write 4 + 2 = ____. Tell the child to enter it on the abacus. Ask: What is the sum? [6] Record the sum. Clear the abacus and write 40 + 20 = ____ below the first equation. Say: Enter 4-ten on your abacus. Add 2-ten. Ask: What is the sum? [6-ten] Record the sum.



Adding 4 and 2.

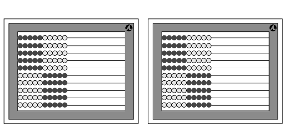


Adding 4-ten and 2-ten.

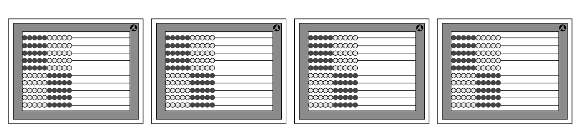
Repeat for the following equations: 2 + 5 = ____ and 20 + 50 = ____. [7, 7-ten] Continue with 8 + 1 = ____ and 80 + 10 = ____ [9, 9-ten] and 7 + 3 = ____ and 70 + 30 = ____. [10, 10-ten] Ask the child if they see any patterns with these equations. [adding tens is like adding ones] Continue with similar sets of equations until the child is confident with the pattern.

Hundreds. Tell the child to enter 10 tens on their abacus. Remind them it has another name, *one hundred*. Ask: How many tens are in 1 hundred? [10] How many ones are in one hundred? [100]

Ask: How could you show 2 hundred? [2 abacuses] Tell the child we will use two abacus tiles, which are pictures of 100 beads entered on the abacus. See the first figure below. Ask them to make 4 hundred. [4 abacus tiles] See the second figure.

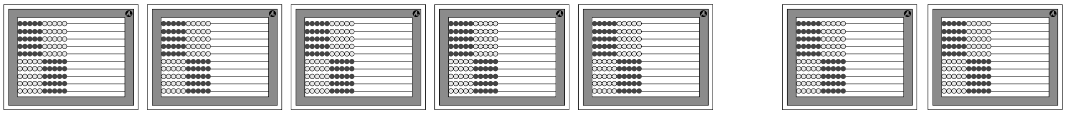


2 hundred with abacus tiles.



4 hundred with abacus tiles.

Ask: Can you make 7 hundred? How could a person tell it was 7 hundred without counting them? [They could group in five and two.] See the figure below. Continue with 8 hundred and 10 hundred. Tell the child that the other name for 10 hundred is *one thousand*.



7 hundred with abacus tiles.

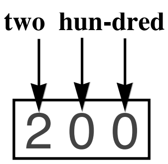
Adding hundreds. Write 3 + 3 = ____. Ask: What is the sum? [6] Use the abacus if needed. Record the sum. Write 30 + 30 = ____ below the first equation. Ask: What is the sum? [6-ten] Again, use the abacus if needed. Ask: Do you think there is a pattern with the hundreds?

Write 300 + 300 = ____ below the second equation. Tell the child to lay out three hundred with the abacus tiles, then add another three hundred. Make sure they group the sum in fives. [600]

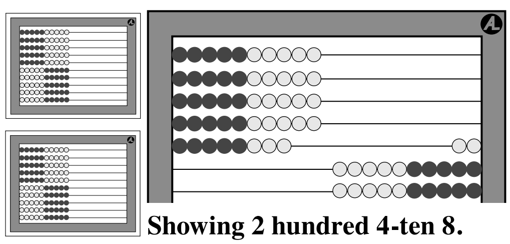
Repeat for the following equations: 4 + 5 = ____, 40 + 50 = ____ and 400 + 500 = ____. [9, 9-ten, 9 hundred] Record the sums. Continue with 2 + 6 = ____, 20 + 60 = ____ and 200 + 600 = ____, [8, 8-ten, 8 hundred] and 5 + 3 = ____, 50 + 30 = ____ and 500 + 300 = ____. [8, 8-ten, 8 hundred]

Ask: Do you see any patterns with these equations? [adding hundreds is like adding tens and ones] Continue with similar sets of equations until the child is confident with the pattern.

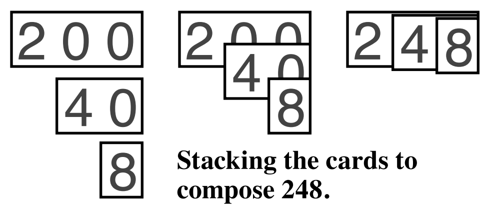
Building hundreds. Show two abacus tiles and ask: How much is this? [2 hundred] Show the 200 place-value card and say: This is how we write 200. Point to the 2 while saying two; point to the first 0 while saying "hun"; and point to the second 0 while saying "dred." See the first figure below. Tell them this number has three syllables and three digits: 2, 0, and 0.



Relating the words "2 hundred" to the digits.



Showing 2 hundred 4-ten 8.



Stacking the cards to compose 248.

Ask the child to enter 4-ten 8 on the abacus and set it next to the two abacus tiles. See the second figure above. Ask: How much is shown altogether? [2 hundred 4-ten 8] Tell them to find the place-value cards for the quantity. Demonstrate how to stack the cards. See the third figure above.

Ask: How many digits do you need after the 7 to write 700? [2] How many digits do you need after the 7 to write 70? [1] How many digits do you need after the 7 to write 7? [0]

Game 5 Can You Find—Level 2

Like the previous day's game, the object of this game is to find the place-value cards for the stated number. This game will now include the place-value cards for the hundreds.

Lay out the place-value cards from 1 through 9, 10 through 90, 100 through 900 face up in no particular order. Explain to the child that you are going to say a quantity for which they are to find the cards.

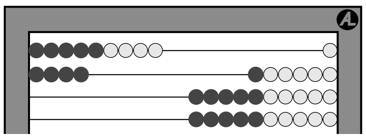
Here are suggested numbers to say: Can you find 4-ten 3? Can you find 4 hundred 8-ten 6? 1 hundred 4? 5-ten 7? Continue using the math way of number naming when calling the numbers: 629? 760? 215? 998? 371? 502? 830? Using these numbers, all the cards will be picked up by the end of the game. Play the game again if the child is interested or needs more practice.

DAY 11 - Make Ten Strategy

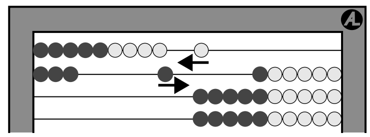
Needed Materials. AL Abacus, Basic Number card deck, and Math Balance

> *NOTE: If a child needs more time to work with a concept, stay on a lesson and work through the material again then replay the game. Modify the pace of the lessons to fit the child.*
>
> *Remember, games are the application for newly learned knowledge. Games provide practice so a concept can become internalized and truly understood. Games also provide a positive environment and experience.*
>
> *Today's lesson will provide a strategy to add 9 to a number. The next day's lesson will extend this strategy of adding 8 to a number.*

Make Ten Strategy. Say: There is another addition strategy, the Make Ten Strategy, that can be used for adding. Enter 9 on the first wire and 4 on the next wire. See the first figure below.



9 + 4 = ___



9 + 4 = 10 + 3 = 13

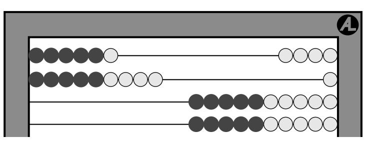
Ask: How could you change the 9 into a 10 while keeping the total quantity of beads the same? Give the child time to think, then say: To have 10 beads on the first wire, trade 1 bead from the second wire for 1 bead on the first wire.

> *NOTE: Make sure the child trades the two beads at the same time. Using two hands, moving in opposite directions, provides a physical sense of equality when making the trade.*
>
> *Some children find it beneficial to say "ready" as they put their fingers on the two beads to be traded, "set" as they prepare to move the beads, and "go" as they make the trade.*

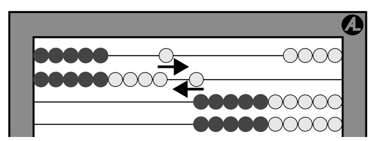
Tell the child to use their left hand to move the bead on the second wire while simultaneously using the right hand to move the bead on the first wire, giving a sum of 1-ten 3 or 13. See above.

Tell the child to find 9 + 7. [1-ten 6 or 16] Repeat for 9 + 5. [1-ten 4 or 14]

Tell the child to find 6 + 9. [1-ten 5 or 15] Ask: Did it matter if the 9 was on the first or second row? [no] See the figures below.



6 + 9 = ___



6 + 9 = 10 + 5 = 15

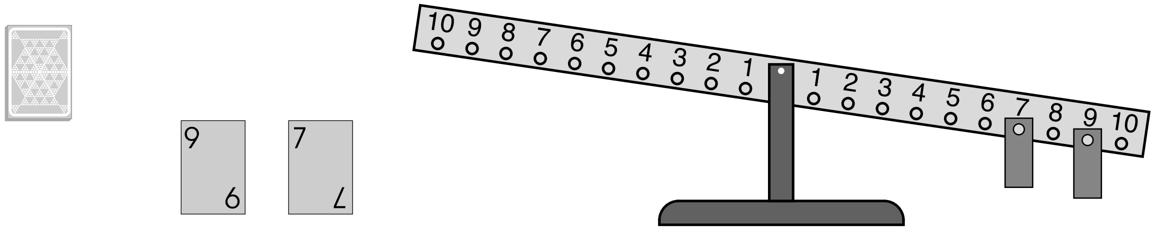
Continue with 8 + 9, [1-ten 7 or 17] 9 + 9, [1-ten 8 or 18] and 9 + 3. [1-ten 2 or 12]

Once the child has done several trades, challenge them to move the beads in their mind rather than physically moving the beads.

> *NOTE: If the child cannot "see" the abacus mentally, have them continue to use the physical abacus until they build a mental model. Gently encourage the mental abacus at a later time.*
>
> *Some children may be so relieved to have a method to find sums with the abacus that they may appear to be unwilling to release the physical abacus. Encourage them to lay the abacus in front of them, but not to touch it, rather to move the beads in their mind. Other children will prefer to have a "pretend" abacus, then use their actual fingers to move the imaginary beads to find an answer. Either scenario will allow for transition to a mental model.*

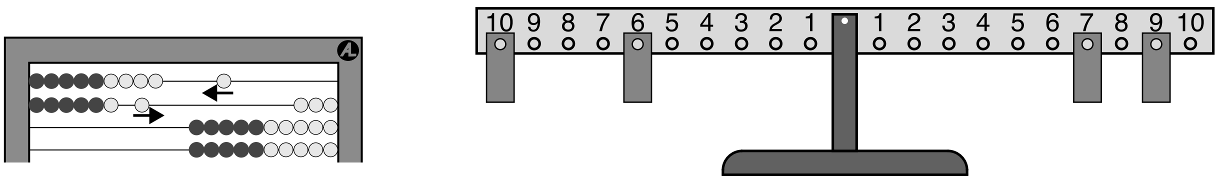
Math balance activity. Place about 15 to 20 cards face down, without any 0 cards. Pull one 9-card from the card deck and lay it face up. Ask the child to put a weight on the 9-peg on the right side. Next turn over a card and put a weight on the right-side peg that corresponds with that card. Tell the child to use their abacus to find the sum. Finally, tell them to put one weight on the left 10-peg and a second weight where needed to balance the equation.

For example, if the card turned over is a 7, they place the weights on the right 7-peg and 9-peg. See the figure below.



9 + 7 with the cards and entered on the math balance.

Using the abacus, check that the sum of 9 and 7 is 1-ten 6 or 16. Put one weight on the left 10-peg and a second weight on the 6-peg to verify the sum. Ask the child to state the equation.



16 = 9 + 7 on the abacus and on the math balance.

Continue with this activity until all the cards are used.

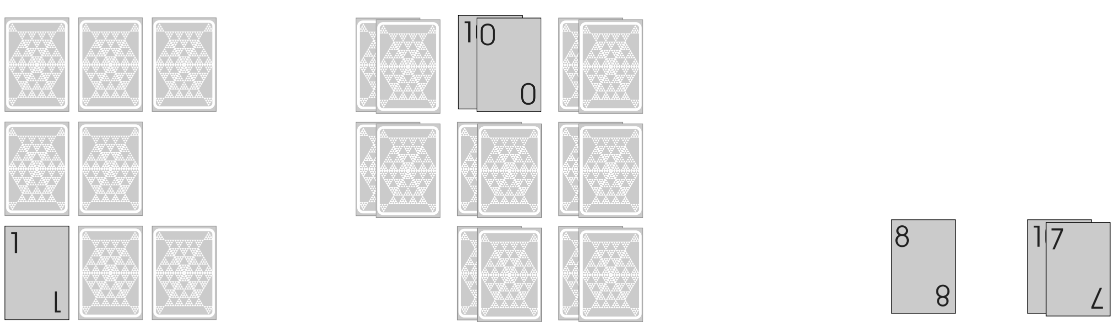
Game 11 Nines Memory

This two-person memory game focuses on adding 9 to a number. This will continue to build the child’s skills and confidence.

From the basic number cards use the following: one 0, two each of 1 through 8, one 9, and nine 10s.

On the left side of the playing area, lay out one each of cards 1 through 9 in random order face down in three rows of three. On the right, lay out cards 0 through 8 in random order face down. Lay a 10-card face down on top of each of the cards on the right; pairs create a 2-digit number.

The players will add 9 to the cards on the left. The pairs of cards on the right will form the sums. The sum of 10 will be formed by the 10-card with the 0-card overlaid. See below.



Play this game like a standard memory game. A player turns over a card on the left, adds 9 to it, says the equation aloud, then seeks the sum among the pairs on the right. If they find a match, they gather the cards and take another turn. If they do not find a match, the cards are returned face down and the next player takes a turn. The winner is the player who collects the most sets.

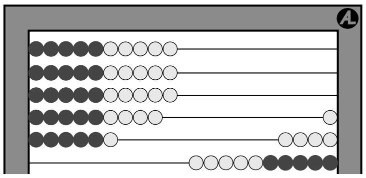
NOTE: Some children may benefit from a small note with “+ 9 =” between the two sets of cards.

DAY 16 - Adding 9s in Higher Decades

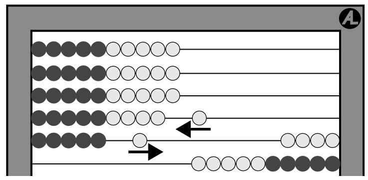
Needed Materials. AL Abacus, Basic Number card deck, and Math Balance

Review adding 9 to a number. Ask: How could you add 9 and 6? [Using the Make Ten Strategy, change the 9 to a 10 by taking 1 from the 6 to get 10 and 5, which is 15.] Encourage them to use the abacus if needed. Ask: What is 4 + 9? [13] What is 9 + 8? [17] Write some more equations and ask the child to give the sums.

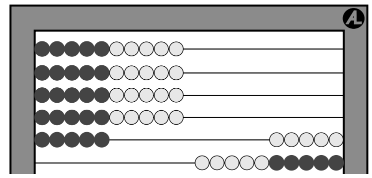
Adding with 9s in higher decades (tens). Write 39 + 6 = ____ and ask the child to enter the two numbers on the abacus. Then ask them to find the sum on their abacus. [Take 1 from 6 to change the 39 to 40, so the sum is 45.] See the figures below.



Entering 39 + 6.



Taking 1 from 6 and giving to 9.



Seeing the sum of 45.

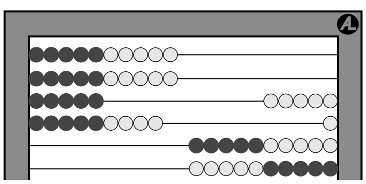
NOTE: Children (and adults) under stress will often revert to old methods, even if those methods and procedures are not effective or productive. Sometimes the pressure is from an outside source, sometimes it is an internal fear or insecurity, or sometimes it is the anticipation of failure.

If a child feels anxiety with these equations and is wanting to revert to finger counting or another inefficient method to find the sum, tell them to take a deep breath. Walk them slowly through the steps. Enter 39. Enter 6 on the next wire. Make ten by trading. Read the sum. Success!

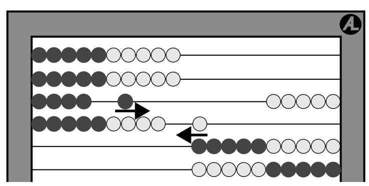
Repeat for 89 + 7, [96] 79 + 5, [84] 19 + 8, [27] 29 + 5, [34] and 49 + 3. [52] Continue with additional equations until the child is comfortable and confident with this process.

NOTE: Some children will use the abacus for every equation and others will begin to use a mental abacus. If the child struggles, refer them to the abacus. The mental abacus will come with continued use of the abacus and when they are comfortable with the process.

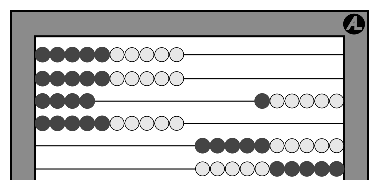
Adding 9s to two-digit numbers. Next write 25 + 9 and have the child enter it on the abacus. See the first figure below. Tell the child to use the Make Ten Strategy to take 1 from 5 and give it to the 9 to find the sum. See the second figure.



Entering 25 + 9.



Taking 1 from 5 and giving to 9.

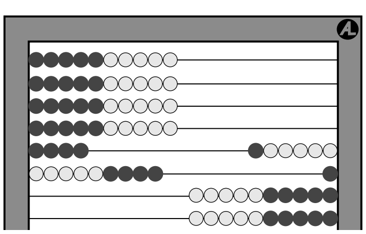


Seeing the sum of 3-ten 4 or 34.

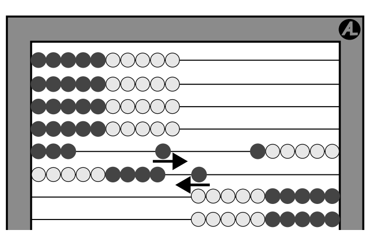
The sum can be seen as 3-ten 4 or 34. See the third figure above.

Some children may not immediately see the sum because the tens are separated by the 4 beads. If needed, ask: How many rows of ten do you see? [3 rows] So what is the sum? [3-ten 4 or 34]

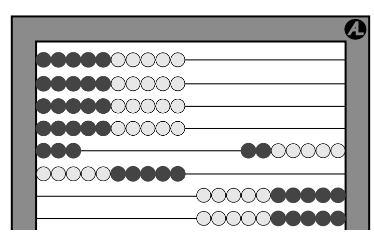
Now write 44 + 9 and have the child find the sum. [53] See the figures below.



Entering 44 + 9.



Taking 1 from 4 and giving to 9.



Seeing the sum of 5-ten 3 or 53.

Repeat for 78 + 9, [87] 66 + 9, [75] 16 + 9, [25] 89 + 9, [98] and 53 + 9. [62]

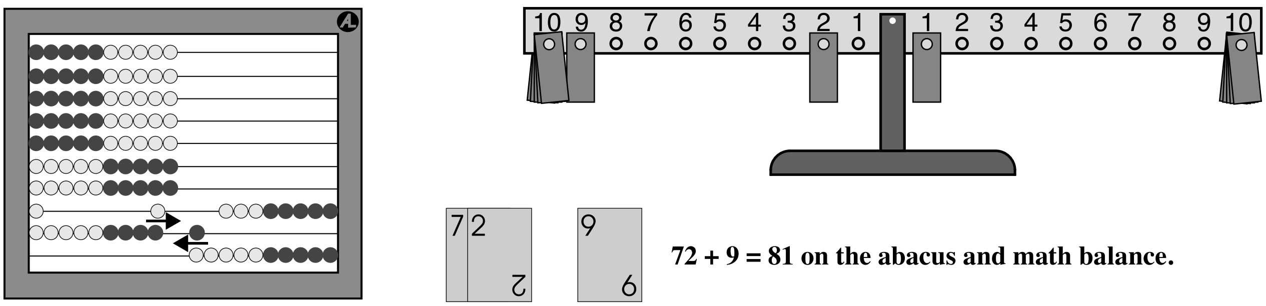
Math balance activity. Lay one 9-card face up. Remove the rest of the 9s and all the 10s from the card deck, then set aside. Shuffle and place about 30 to 40 cards face down. Have the child turn over two cards to create a two-digit number by overlapping the cards.

Ask the child to put weights on the math balance to represent the two-digit number plus the addend 9. Tell the child to use their abacus to find the sum. Finally, tell them to put the weights on the other side of the balance to represent the sum.

NOTE: Both the front and back of the math balance arm can be used to place the weights.

For example, the cards turned over are a 7 and 2 to become 72. Ask the child to enter 72 on the math balance. If they are unsure how to do this, remind them to say the number the math way: 7-ten 2. Guide them to place 7 weights on the 10-peg and 1 weight on the 2-peg.

Add a weight on the 9-peg. Find the sum on the abacus [81] and enter it on the math balance by placing 8 weights on the 10-peg and 1 weight on the 1-peg. See figure below.



72 + 9 = 81 on the abacus and math balance.

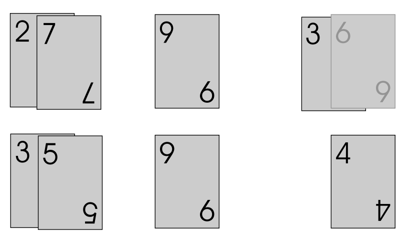
Continue with this activity until all the cards are used.

Game 16 9s Mental Addition

This game helps the players with their mental addition by adding 9 to a two-digit number. Use the AL Abacus when needed.

This two to four person game will use all the basic number card deck without the 10s. Find four 9s to be used as the addends. Shuffle and deal five cards from the stock to each player. Following each turn, the player draws from the stock until he has five cards in hand.

Start two rows with a two-digit number formed using cards from the stock. Place a 9-card slightly to the right as an addend for each row. See the figure.



The first row of three cards forms 27 + 9 and the second row of cards forms 35 + 9. Sums of 36 and 44 are being created.

The object of the game is to complete a row using cards from the player's hand to form the sum of the two numbers. The player who completes the row collects the cards, leaving the 9 addend in place. The winner is the player who collects the most cards.

Players take turns playing one or two cards. Either digit of the sum may be played as it is available, and a player may lay down one or two cards in any row during their turn. If a player is unable to play a card in any row, he starts another row with two cards from the stock and a card from the 9s stack. If he still cannot play, he skips his turn. If four rows are laid out and he is unable to play a card from his hand, he may replace all or some of his cards from the stock and his turn is ended.

Always have between two and four rows available to play. The game is over when the cards are exhausted, and no more cards can be played.

DAY 24 – Adding with Base-10 Picture Cards

Needed Materials. Base-10 Picture cards, Place-value cards, and paper and pencil or dry erase board and marker

Review. Show the child one ten base-10 picture card and ask: Suppose I had 10 of these cards. How much would I have? [100] Now suppose I had 60 of these cards. How much would I have? [600] Ask the child to explain. [Each 10-ten is 100, so 60-ten is 600.] Show the 600 place-value card and ask: Is it the same? [yes] Why? [it shows 60-ten or 6 hundred]

Ask: When do you need to trade base-10 cards? [when you have 10 or more of the same card]

Adding. Tell the child the following story:

> City planners are building a swimming pool and need to know how many children live in the surrounding towns of Braddock, Kintyre, and Temvik. Braddock has 2697 children. Kintyre has 3986 children and Temvik has 1449 children.

NOTE: These three towns are considered ghost towns in North Dakota, although all have a few residents remaining.

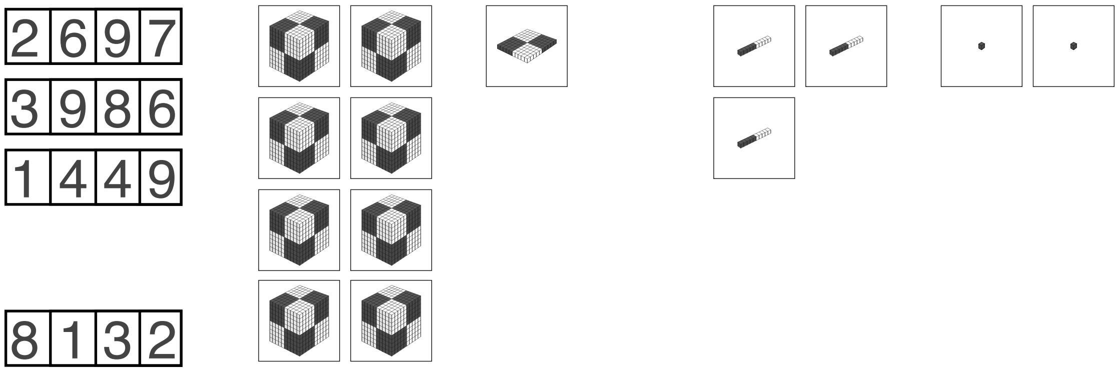
Tell the child to construct the numbers with their place-value cards and collect and lay out the corresponding base-10 cards. Reread the story as needed.

Have the child lay the three sets of place-value cards vertically as shown below. Ask them to combine the base-10 cards.

6 thousands, 19 hundreds, 21 tens, and 22 ones.

Tell the child to make trades whenever they have ten or more cards of the same denomination, taking the 10 cards to the bank where they make the appropriate trade. If needed, remind the child to gather groups of ten starting from the bottom of the columns.

After the trading has been completed, have them compose the sum with the place-value cards. The results are shown below.



The sum after trading.

Summarization. Ask: How many children are in the three towns? [8132] Does this answer make sense? Tell the child to write the numbers and the sum on the paper or dry erase board. Then ask them to explain how they found the answer.

> *NOTE: The act of explaining something is another step in the learning process. A child may know the answer to an equation, but explaining how they found the answer adds an additional layer to their understanding. The act of organizing and expressing their thoughts that is important to their understanding.*

More adding. Tell the child the following story:

> Bird watchers count the number of birds they see in their area to help scientists monitor the quantity and location of birds, including changes from habitat, disease, and climate. Over a particular weekend, bird watchers counted 879 robins, 4387 finches, and 2718 swallows. How many birds did the watchers count?

Tell the child to solve the problem using the same process with the place-value cards and corresponding base-10 cards. [7984 birds]

Game 24 Banking Game—Level 2

This one or two person game helps the players practice their four-digit addition skills. They will need the place-value cards and the base-10 cards to compose the numbers, then negotiate trades with the base-10 cards.

Using the place-value cards, have the child create two four-digit numbers.

> *NOTE: Because there are only 9 thousands base-10 cards, guide the child to choose numbers that will have a sum less than 10,000.*

Tell the child to lay out the corresponding base-10 cards, then add the base-10 cards together. When looking at gathering cards for trades, remind them to start at the bottom of the column. Also, there is no need to start at either the left or right of the layout; trades can be made in any column as they are discovered.

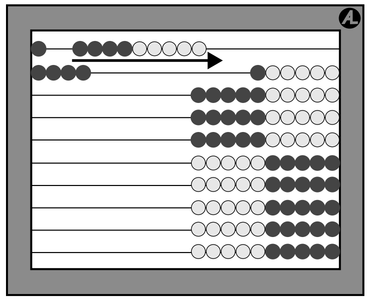
If two players are playing, have them switch the banker and adding roles. Make sure the banker verifies that all trades are "fair" and that ten of one denomination is being traded for one of the next higher denomination.

DAY 37 – Taking All from Ten Strategy

Needed Materials. Paper and pencil or dry erase board and marker, AL Abacus, and Basic Number card deck

Subtracting 9. Write 14 – 9 = __ for the child to see. Tell them to enter 14 on the abacus. Ask them how they could find the difference on their abacus. [Take 4 from the 4 on the second row and 5 from the 10, leaving 5 as the difference.] Say: There is another way to subtract 9.

Have the child enter 14 on the abacus again. Now tell them to subtract 9 from the 10 on the top row, as shown below. Then ask: What is left? [1 and 4, which is 5] Ask the child to write the equation: 14 – 9 = 5.



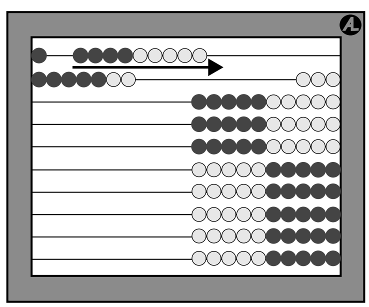
Subtracting 14 – 9 by taking 9 from the 10.

Ask: What do you think of this new strategy? Is this strategy easier than the Taking Part from Ten strategy? Say: This new strategy is called Taking All from Ten.

NOTE: In the Middle Ages, people did not bother to memorize subtraction facts past 10 because they used the Taking All from Ten Strategy.

Taking All from Ten strategy. Ask the child to solve 17 – 9 using the new strategy. See the figure below.

NOTE: Do not tell the child that subtracting 9 is one more than the number in the ones place of the multi-digit number. For example, 17 – 9 is one more than 7, which is 8. Let the child make that discovery themselves and use it when it makes sense to them.

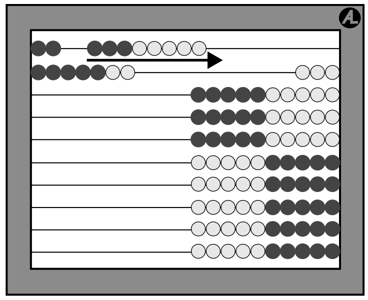


Subtracting 17 – 9 by taking 9 from the 10.

Tell the child to write the equation: 17 – 9 = 8.

Repeat for 12 – 9, [3] 15 – 9, [6] 16 – 9, [7] 13 – 9, [4] 18 – 9, [9] and 11 – 9. [2] Ask the child to write all of the equations.

Subtracting 8. Ask: How could you use the Taking All From Ten strategy to find 17 – 8? [by subtracting 8 from the 10] What is left? [7 + 2 = 9] Ask them to demonstrate it on the abacus. See the figure below. Ask the child to write the equation: 17 – 8 = 9.



Subtracting 17 – 8 by taking 8 from the 10.

Repeat for 16 – 8, [8] 15 – 8, [7] 14 – 8, [6] 13 – 8, [5] 12 – 8, [4] and 11 – 8. [3] Ask the child to write all of the equations.

Challenge. Ask: How could you use the Taking All From Ten strategy to find 12 – 7? [Take 7 from 10 and add 3 + 2 = 5.] How could you use the strategy to find 13 – 6? [4 + 3 = 7]

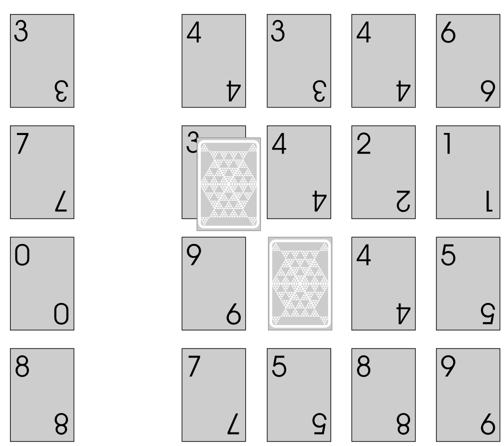
Game 37 Subtraction Bingo

Children who like to play Bingo will enjoy this game. This two to four player game gives practice in using all the subtraction facts. Use the full deck of basic number cards, without the 10s.

Deal each player 20 cards; the remaining cards form the stock. Each player takes 16 of their cards and lays them face up in a four-by-four rectangle which will become the differences. Place the last four cards to the left of the rectangle, a short distance away, as shown below. These cards will be the number to subtract from.

The goal of the game is to be the first player to cover a row, column, diagonal, or four corners of the rectangle with cards. There may be ties.

Each player takes a card at the same time and subtracts it from a number on the left of their grid. A one in the tens place is presumed to be present when needed. For example, 3 – 4 will become 13 – 4. If the difference is found in that row, the player covers it by laying the drawn card on top, face down. If a difference cannot be found in any of the rows, the card is discarded.



Refer to the figure on the right. For example, if the player draws a 4, no difference exists in the first row because 13 – 4 = 9 and no 9s are available. However, in the second row, 7 – 4 is 3 and a 3-card is available. Place the 4-card face down to cover the 3. No changes are permitted once a card is laid down.

Players draw a card from the stock at the same time, then check their grid and place cards as they can. Repeat until a player covers a row, column, diagonal, or the corners. If the stock becomes exhausted, use the discard pile.

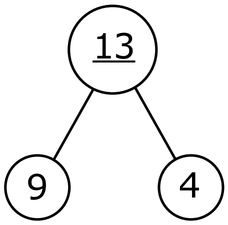
After the game is complete, players can verify their equations by picking up a face-down card along with the card below it and adding the two together. The sum should equal the card at the far left of that row (subtracting 10, if necessary). In the example, the 4-card is covering the 3-card, 4 + 3 = 7.

DAY 45 – Problems Using Subtraction

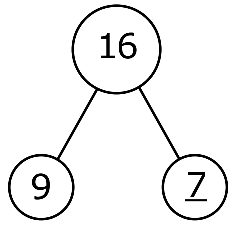
Needed Materials. Paper and pencil or dry erase board and marker, AL Abacus, and Basic Number card deck

Reviewing part-whole circle sets. Draw a part-whole circle set as shown in the first figure below. Write 9 and 4 in the parts and ask the child: What goes in the large circle, the whole? [13] How do you know that? [add the parts]

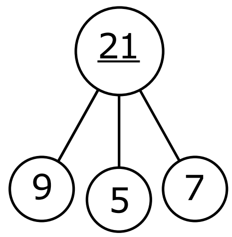
Change the 13 to 16 and erase the 4. Ask: What is the missing part? [7] How did you find the missing part? [subtract the part from the whole; 16 – 9 = 7] See the second figure below.



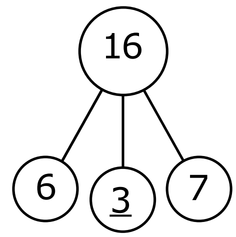
Adding the parts to find the whole.



Subtracting from the whole to find one of the parts.



Adding the parts to find the whole.



Adding the parts to find the whole.

Erase the 16. Add another part-circle and write 5 in it as shown in the third figure above. Ask: What is the whole? [21] How did you find it? [add the parts]

Finally, write 16 in the whole-circle, change the 9 to 6 and erase the middle part, the 5. See the last figure above. Ask: What is the missing part? [3] How did you find it? [add the 6 and 7 to get 13 and subtract that from 16 to get 3, or subtract 6 from 16 to get 10 and subtract 7 from 10 to get 3]

Problem 1. Read the following story to the child:

Hayes has 75¢. He buys 2 pencils, each costs 29¢. How much change does Hayes receive?

Give the child a few minutes to think about the problem. Then ask them to share the solution with you. [17¢] Two possible methods to find the solutions are:

$$\begin{array}{r} 75¢ \\ -\ 29¢ \\ \hline 46¢ \end{array} \qquad \begin{array}{r} 46¢ \\ -\ 29¢ \\ \hline 17¢ \end{array} \qquad\qquad \begin{array}{r} 29¢ \\ +\ 29¢ \\ \hline 58¢ \end{array} \qquad \begin{array}{r} 75¢ \\ -\ 58¢ \\ \hline 17¢ \end{array}$$

In the first solution, 29¢ is subtracted twice. In the second solution, the cost of the pencils is found by adding 29 + 29 and then subtracting the total, 58¢, from the starting amount, 75¢.

NOTE: If the child is unsure with the trading aspect of the subtraction problem, encourage them to use their abacus. Although two methods shown here, this problem can be solved a number of different ways. For example, 29 is close to 30 and 30 + 30 = 60. Then 75 – 60 = 15 and 15 plus the "borrowed" 2 is 17¢.

Problem 2. Read the following story to the child:

Makenna is saving money to buy a Mother's Day gift, which costs $10.69. She had saved $3.19 and earned $4.86 more. How much more money does Makenna need? [$2.64]

Two possible solutions are shown below.

$$\begin{array}{r} \$3.19 \\ +\ \ 4.86 \\ \hline \$8.05 \end{array} \qquad \begin{array}{r} \$10.69 \\ -\ \ \ 8.05 \\ \hline \$2.64 \end{array} \qquad\qquad \begin{array}{r} \$10.69 \\ -\ 3.19 \\ \hline \$7.50 \end{array} \qquad \begin{array}{r} \$7.50 \\ -\ 4.86 \\ \hline \$2.64 \end{array}$$

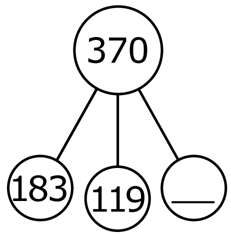
Problem 3. Read the following story to the child:

The total distance from Windsor, Canada to Toronto is 370 km with London and Hamilton between the two. The distance from Windsor to London is 183 km. The distance from London to Hamilton is 119 km. What is the distance from Hamilton to Toronto? [68 km]

If the child is interested, tell them that Canada uses metric measurements for distance. One hundred kilometers is about 62 miles.

Have the child draw a diagram as shown below. Reread the story and tell them to write the distances on the figure as they hear them.

Ask them to think about what they know and what they need to find. Tell them to use a part-whole circle set as shown below. Ask: Why does the part-whole circle set have three parts? [The distance from Windsor to Toronto has three parts.] Give them time to think about the problem, find the answer, [68] and write an equation. See below.



183 + 119 + ____ = 370

370 – 183 – 119 = ____

370 – (183 + 119) = ____

The part-whole circle set and some possible equations.

Ask them to explain their solution. Ask if their equation could be written another way. Several options are shown above.

One way they may have solved it is to add 183 and 119 [302] and go up to 370 to get 68 km. This follows the first equation shown above.

Another way is to subtract 370 – 183 [187] and then subtract 187 – 119 to get 68 km. This follows the second equation.

Still another way is to add 183 and 119 [302] and then subtract that from 370 to get 68 km. This follows solving the third equation shown above.

Ask: Does your answer sound reasonable? Does it make sense that Hamilton is 68 km from Toronto? [yes]

Game 45 Repeated Subtraction Solitaire—Level 1

This self-checking solitaire game provides the player with subtraction practice. Use the basic number cards 0 through 9 to build the starting number. Then pull out a 0-card to be used in the ones place.

The player builds a 4-digit minuend number using the 0-card and three more cards from the deck. Arrange the cards to create a number less than 5000 with the 0-card in the ones place. Next, to create the subtrahend, use the first three digits from the number and double it.

For example, if the 4-digit number is 3590, the first three digits, 359, are doubled to create the subtrahend of 718. So the 3-digit number to subtract will be 718.

The object of the game is to subtract the 3-digit number from the 4-digit number until 0 is reached. Have the player record their work on paper. Discuss the number of times they subtracted to get to 0. [5]

Repeat the procedure with a new 4-digit number and follow the process until they get to zero.

$$\begin{array}{r} 3590 \\ \underline{-\ 718} \\ 2872 \\ \underline{-\ 718} \\ 2154 \\ \underline{-\ 718} \\ 1436 \\ \underline{-\ 718} \\ 718 \\ \underline{-\ 718} \\ 0 \end{array}$$