RightStart™ Tutoring

by Kathleen Cotter Clayton
and Joan A. Cotter, Ph.D.

MULTIPLICATION AND DIVISION
BOOK TWO

Activities for Learning, Inc.

A special thank you to Rachel Anderson, Maren Ehley, Teresa Foltin, Debbie Oberste, and Beth Reid for their contributions.

Cover design by Maren Ehley.

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INTRODUCTION

Welcome! This manual is about understanding multi-digit multiplication, short division, and long division with the end goal of becoming confident with these processes and applications. It is intended for those who have a weak or incomplete grasp of multi-digit multiplication and multi-digit division, and who are two or more grade levels behind.

Multiplication has been the mathematical downfall for many students, not so much because of the algorithms, but because they are required to memorize 100 basic facts, forwards and backwards. When the student has a shaky knowledge of the facts in conjuction within multi-digit algorithms, the collapse of arithmetic is a scary scenario.

Attempts have been made to solve these struggles by focusing on rote memorization of the processes without full comprehension. For many, the burden of memorization is overwhelming, never mind the frequent need to review. Students who have memorization without understanding struggle to apply their skills to new situations. This results in frustration, confusion, and an aversion to math.

On the contrary, we now know that a deep understanding of concepts removes anxiety, lessens the burden of memorizing, makes advanced math easier to grasp, and makes math more enjoyable.

It does not matter if the student is 12 or 112 years old; these lessons will approach multi-digit multiplication and multi-digit division with a new perspective that follows the RightStart Mathematics approach and philosophy.

There are strategies used in this manual that are different from the traditional way multiplication and division are taught. Although they will be explained in greater detail during the appropriate lessons, here is a quick overview.

Multiple Approaches

Multiple approaches will be presented to solve multiplication and division problems. These are not given to confuse a student; rather, they provide options. One strategy might become the student's favorite, but the next day's strategy could be even better. Multiple approaches give the student additional perspectives to expand their understanding.

If a strategy or approach does not resonate with you as the teacher, that does not mean it won't be important to the student. Follow the lessons because it may be critical in helping the student understand.

There will be approaches that are stepping stones to more traditional (and familiar) approaches. Do not skip these steps, as they are intended to build understanding and create a firm foundation to build upon.

The Cotter Abacus will show how and why the traditional algorithm works. It will not become a crutch; rather, the student will physically maneuver the beads on Side 2 of the abacus, making the processes concrete.

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Short division is the gateway to learning long division. This short division process is much easier to understand and learn, giving long division a base to build upon.

So, what is short division? Short division is dividing a number by a single-digit number without writing anything below the dividend. That is, no subtraction steps are recorded. An example is shown below.

$$\begin{array}{r} 3\,4\,6\,4 \\ 2\overline{)6\,9\,{}^{1}2\,8} \end{array}$$

An old arithmetic textbook discussed the importance of teaching long division from the standpoint of short division. In the book, the author McMurry describes how short division provides an understanding of the long division algorithm. He also emphasizes the importance of children understanding the long division process and not merely memorizing a list of rules.

Fifty years later, after the deletion of short division from the curriculum in elementary arithmetic, another author lamented the elimination of short division as an introduction to the long division process. And indeed, they were right. Today, many adults have never heard of short division, and they believe that long division is merely a rote process.

Math Card Games

Many students get overwhelmed with math worksheets. Students who do not understand a concept will not benefit from more and more worksheets. Rather than worksheets or flashcards, games will be used in this manual.

These math card games will allow the student to learn and practice new skills. Games often keep math time enjoyable. The emotions that are experienced while learning are stored along with the knowledge. If a student has an enjoyable time learning, then positive emotions will replace past negative emotions.

A game will be assigned in each lesson. Some are solitaire games, and some are for two or more players. Include other family members or students in the games. Nothing more motivating than a student playing a game against their parent or teacher—and winning!

Game instructions are given in each lesson. Adapt as necessary to fit the student and the situation. For example, turn the games into one-person games or modify them to fit more than one player. Please contact RightStart Math if you need ideas for modifying the games.

The games will hone skills and help the student become more confident and fluid in their thinking. The more games are played, the more the student learns. If a concept is not solid, play the game again. Also, playing previously-played games will allow the student to see their growth and master their facts.

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Summary

The lessons, activities, and games in this program are from the RightStart™ Mathematics curriculum and *Math Card Games, 5th edition*, both written by Dr. Joan A. Cotter, along with some new games. This manual can be used alongside any math program; knowledge of the RightStart™ Mathematics program is not required.

This manual is the teaching guide, and the games and activities create an interesting learning environment. If a student struggles, slow down the lesson, provide additional examples, and concentrate on the activities and games. Make sure they are using the Cotter Abacus.

In these 47 days of lessons, a solid foundation of more advanced multiplication and division will be laid while proceeding step by step to develop a clear understanding. There are no worksheets; rather, daily games will provide practice and review.

This book is Multiplication and Division, Book Two and addresses multi-digit multiplication, short division, and traditional long division. If the student is unsure of the multiplication facts up to 10 × 10 and division of numbers 100 or less, including remainders, consider using RightStart™ Tutoring Multiplication and Division Book One which focuses on those skills and facts.

We believe that through these lessons and games, students will develop a renewed interest in and enjoyment of mathematics, thereby enriching their lives. We also hope many of them will become tomorrow's mathematicians, scientists, and engineers.

We want you and your students to have great success in learning and developing confidence with multiplication and division. Let us know how this tutoring program benefits you and your students. Please share your experience and keep in touch!

Kathleen Cotter Clayton

Joan A. Cotter, Ph.D.

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DAILY LESSONS

Needed Materials

Materials for the day's activities will be identified at the beginning of the lesson. Paper and pencil or a dry erase board and marker may be needed. If an appendix page is referenced, it will be found in the back of the book. Some appendix pages will need to be copied.

The Cotter Abacus will enable the student to build a mental model necessary for concept formation. Even if a student knows a process, it is important they also see it physically on the abacus in order to develop an understanding of the relationships between numbers and the operations that modify them.

Manipulatives are not to be regarded as crutches but rather as tools for learning. In practice, the student will refer to them less and less and finally not at all. Sometimes, just the security of having them nearby helps, even if they are not used.

Activities

This section is the heart of each day's lesson and provides instructions for teaching. Within these instructions, you will guide the students to discovery by asking questions. The expected answers from the student are given in square brackets. [like this]

Avoid talking during problem solving time. Resist the temptation to rephrase the question. This time gives the student an opportunity to think, visualize, and solve a problem. Encourage the student to develop persistence and perseverance. Avoid giving hints or explanations too quickly. Students, and people in general, tend to stop thinking once they hear the answer.

Notes are included in the lessons to help the teacher understand why something is done or not done. These are not directed toward the student, but to provide additional information for the tutor.

Games

Daily games, not worksheets or flashcards, provide practice for the new skills. The games can be played as many times as necessary until proficiency occurs. They are as important to learning math as books are to reading. Reviewing previously-played games lets the student see their progress while reinforcing familiar concepts.

Worksheets

There are no worksheets for this tutoring manual. Practice will come from the games.

Word problems are included thoughout the lessons, with copies in the appendix so the student can follow along or read the problems themselves.

There will be situations where equations need to be written out. Some students may struggle with using paper and pencil yet will find a dry erase board and marker smoother and easier to work with. If you need or want to record work from a dry erase board, take a picture, then save it for your records. Or, if the student finds writing uncomfortable, painful, or difficult, we recommend the teacher becomes the scribe, writing exactly what the student says, even when it's a wrong answer.

THE MATH GAMES

Math card games develop the players' math skills while they play. They will learn and practice the processes as they play, using the manipulatives for support. More importantly, the games give the players an application to practice.

Strategies provided in the daily lessons will give students confidence and independence. What is a simple step to someone who knows multiplication or division often takes additional steps for a struggling learner. The variety of games and activities will support the process. Often, a concept can be learned in more than one way, resulting in different games for the same idea.

Do not hurry to get to the next lesson and game. Frequently, go back to games already learned; the student will often play them with a new perspective. Game Day lessons will review previously learned ideas, although replaying games beyond these lessons is strongly encouraged. Ideally, additional math card games should be played outside of the lesson time.

Description of the Cards

To play the daily games, you need two decks of special cards, which are available from Activities for Learning, Inc. The descriptions are as follows:

Basic Number Cards

These 132 cards are numbered from 0 through 10. There are 12 of each number.

Multiplication Cards and Envelopes

Each card in the multiplication deck corresponds to a number in the multiplication table from 1×1 through 10×10. Thus, it has 100 cards. Some numbers, such as 1, are found only once, and others, such as 6, are repeated multiple times.

Ten envelopes, one for each multiple, are included. These are used to sort the cards into groups matching the pattern on the front of each envelope.

Some find it helpful to have two multiplication decks: one shuffled deck and a second deck sorted into the appropriate envelopes.

The player with learning challenges

Those with learning challenges often find memorization very difficult and paperwork tedious. These games eliminate both issues and give the student a new approach to practicing their new skills. Work in a place free from overwhelming noise and visual distractions. Repeat the games as many times as needed.

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DR. JOAN A. COTTER'S BACKGROUND

Dr. Joan A. Cotter's love of children and their ability to learn goes well with her love of math and her desire to make it understandable and create a successful experience for everyone. Adults who teach the RightStart program often exclaim how much it has helped them better understand mathematics.

Dr. Cotter's educational background includes a Bachelor's in Electrical Engineering, a Master's in Curriculum and Instruction, and a Ph.D. in Mathematics Education. Her research was with elementary students learning mathematics, especially place value.

She earned a Montessori diploma and taught children, ages 3 to 6 years, in her own Montessori school. She also taught middle-school mathematics and tutored special education students. Dr. Cotter wrote the RightStart™ Mathematics program for school and home educators.

An interesting fact that Dr. Cotter finds fascinating is that researchers have recently found that when people discover beauty in math, their brains light up in the same regions as that of artists when they find beauty in art. Understanding math brings out the beauty of math.

Dr. Cotter continues to write and speak across the US and internationally. She lives in Minnesota, where she continues to run the business, Activities for Learning, Inc. Joan and her husband have three adult children and five grandchildren.

If you are intersted in more information on Dr. Cotter's approach to teaching math, go to RightStartMath.com or RightStartClassroom.com.

RightStart™ Tutoring Multiplication and Division
Book Two
Table of Contents

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RightStart™ Tutoring Multiplication and Division: Book Two

Table of Contents

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DAY 11 - Two-Digit Multiplication

Needed materials. Paper and pencil or dry erase board and marker, Cotter Abacus, Multiplication cards in their envelopes, and Basic Number card deck

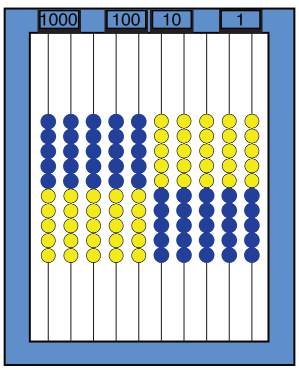
Review. Say: So far, we have worked on multiplying by ten and multiples of ten, such as 20, 30, and 40. We have also learned how to break a number down, multiply the parts, and then add them together to find the product. For example, 24 × 3 is the same as 20 × 3 plus 4 × 3. In the last lesson, we learned how to multiply a two-digit number on side 2 of the Cotter Abacus. Today, we are going to write down the process as we practice two-digit multiplication on side 2 of the abacus.

Recording 2-digit multiplication. Tell the student to write the equation 24 × 3 vertically:

24
× 3

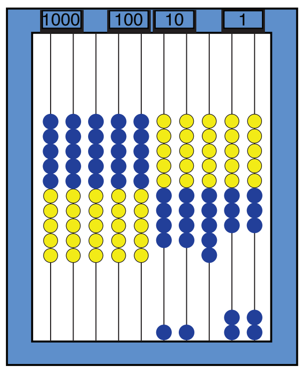
Say: Move the beads halfway up the abacus like we did in the previous lesson. See the first figure below. Next, enter 24 on the bottom, as shown in the second figure. Say: We will start with the ones place, on the **right**, and multiply 4 × 3. Tell the student to move the 4 beads up to the center, just before entering 12 on the top, as shown in the third figure. Then, tell them to write the *partial product*, 12, below the line of the equation. See the third figure.

NOTE: As the student writes out each step of the multi-digit multiplication process, they develop understanding, and it prepares them for the traditional algorithm, which will be taught in a future lesson. Do not worry about carrying, or the carries, at this time.



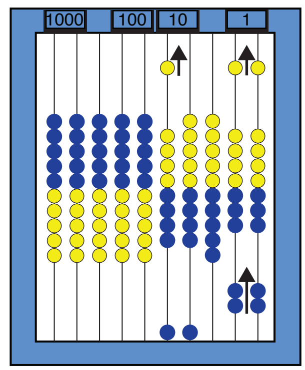
24
× 3

Abacus prepared



24
× 3

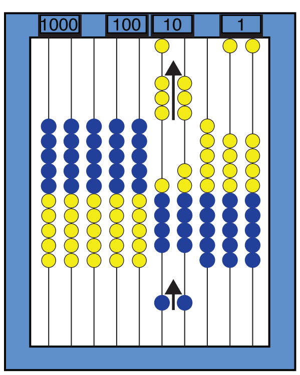
24 entered



24
× **3**
12

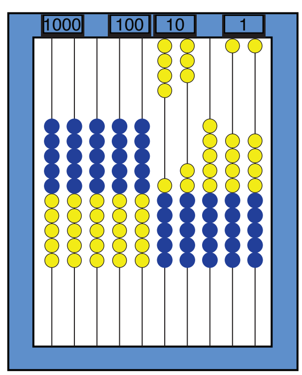
4 × 3 = 12

Say: Next, multiply 20 × 3. Tell them to enter the multiplication quantities on the abacus first. Ask: What is the partial product? [60] Write it below the first partial product, 12. See the first figure below.



24
× **3**
12
60

20 × 3 = 60



24
× 3
12
60
72

The final product

Ask: What is the final product shown on the abacus? [72] Look at the written equation; how do you get the final product? [add the partial products] See the last figure displaying the solution, 72.

Another equation. Tell the student to complete a second expression, 49 × 7, using the same process. If needed, remind them to start with the ones. The solution is shown on the next page.

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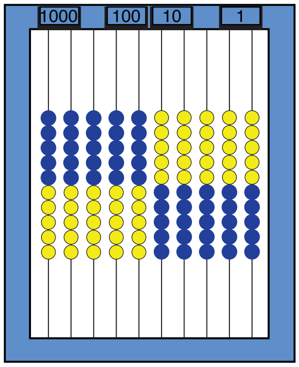
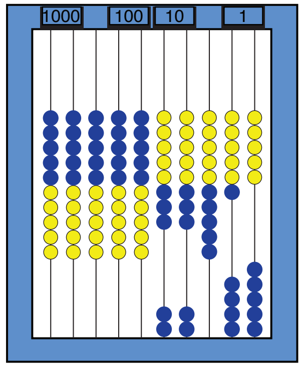


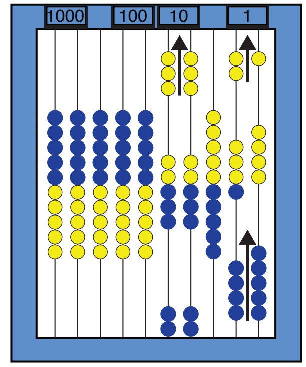
49
× 7

Abacus prepared



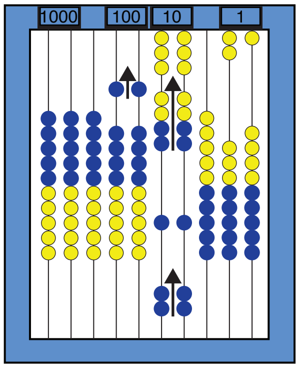
49
× 7

49 entered



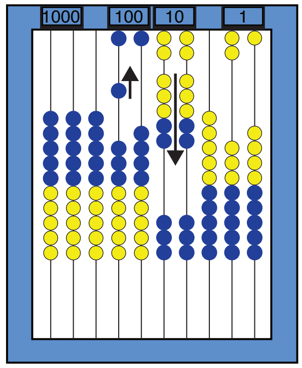
49
× **7**
63

9 × 7 = 63



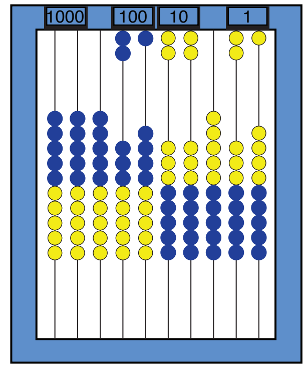
49
× **7**
63
280

40 × 7 = 280



49
× 7
63
280

Trading

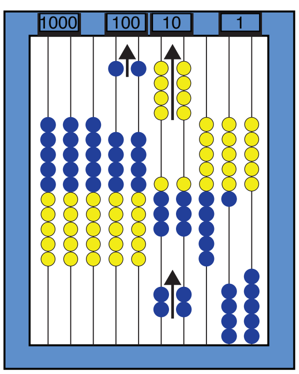


49
× 7
63
280
343

The final product

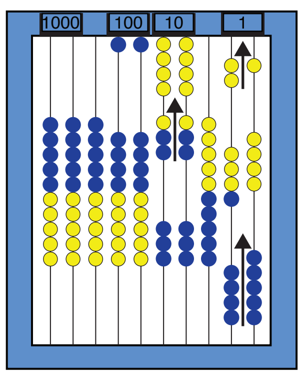
Curious question. Say: When we did the equations on the abacus previously, we multiplied the tens first, then the ones. Ask: Do you think the partial products can be recorded in that order? Will it change the solution? [answers will vary]

Tell the student to complete the same expression, 49 × 7, using the abacus and recording each step, this time starting with the **tens**. The solution is shown below.



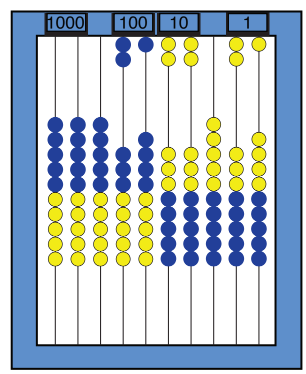
49
× **7**
280

40 × 7 = 280



49
× **7**
280
63

9 × 7 = 63



49
× 7
280
63
343

The final product

Ask: Did the product change when multiplying the tens first? [no] Does it matter whether the ones or the tens is multiplied first? [no] Say: Although some will use this approach when solving an equation mentally, the process is traditionally done on paper by starting with the ones. It is also easier to start with the ones when carries are involved.

Game: Would You Rather Have...?—Level 1

NOTE: For today's game, use two sets of multiplication cards, different from the previous game. Encourage the players to create new items to collect. Some players may wish to create silly or extravagant items, such as cars, puppies, or horses. Play the game two or three times, changing the items to collect each time. Game instructions are found on Day 10.

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DAY 14 – Game Day

Needed materials. Multiplication cards in their envelopes, Cotter Abacus, Short Multiplication table (Appendix p. 1), paper and pencil or dry erase board and marker, and Basic Number card deck

Games. Tell the student that today is another game day! This is a day to explore and apply what they have learned so far.

> *NOTE: Remember to keep math time pleasant. Take time to help the student enjoy the games, practice new skills, and relish newfound confidence. Games should be played often, as part of the lesson and as "homework" after the lesson.*

Say: Remember, replaying math card games helps you improve your skills and increase your speed when finding solutions. A game you found challenging a few days ago may be easier today because you know more now. Playing these games will be well worth the time and effort as you continue to experience growth and increased skills.

Tell them they will play a game to keep the multiplication facts fresh, and then they will play a game that will work on the procedures of multiplying 3- and 4-digit numbers.

Game: Lowest in the Corners—6, 7, 8, and 9

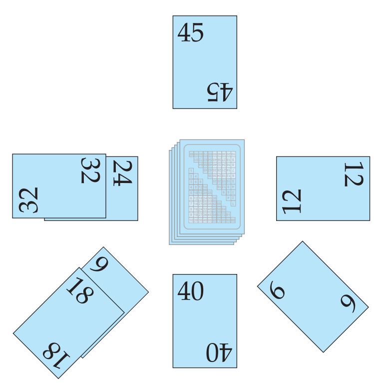
> *NOTE: This game helps the student review the multiplication facts for the numbers that are often considered more challenging. Encourage players to use their abacus or the Short Multiplication table as needed. This will help them with the facts they might be struggling with.*

Find the ten multiples of 6, 7, 8, and 9, a total of 40 cards, and shuffle them together. Deal each player seven cards. The remaining cards form the stock. Then, place the top four cards from the stock face up around the stock, one on each side.

The players begin each turn by taking one card from the stock and playing as many cards from their hand as possible according to the following rules:

1. The sets are built in ascending order on either the initial cards or at the corners. The first card played in a corner must be the lowest card of one of the sets, a 6, 7, 8, or 9-card.
2. A card or group of cards may be moved to any other position, provided they follow the first rule.

   NOTE: To assist in determining which set is being built, subtract any two adjacent numbers. For example, 32 – 24 is 8; therefore, this is the set of 8s.
3. As well as playing on cards already laid down, the player may fill in any empty spaces with one or more cards from their hand.
4. The same number may not be used twice for the same set. In other words, since one 24-card is used in the 8s series, the second 24-card may only be used in the 6s series.
5. Players must play their cards when possible, saying the multiplication equation, for example, 9 × 2 = 18.



Game in progress. The 12-card can be placed on the 6-card.

The winner is the first player to play all the cards from their hand.

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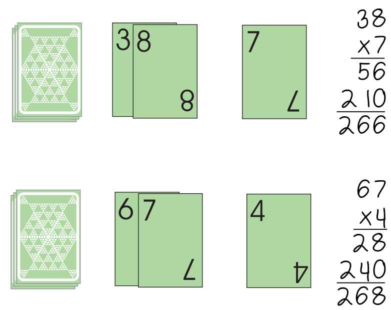
Turbo Multiplication War

The objective of this two-person game is to give the players practice multiplying multi-digit numbers. Play using about half the deck of basic number cards. Shuffle the cards. Create two even stacks by comparing their heights and giving one stack to each player. Each player will also need paper and pencil or dry erase board and marker. Some players may also want to use their abacus.

Both players take the top three cards from their stacks, set them face up, and slide the first two cards together to create a 2-digit number. The third card will be the multiplier. Each player multiplies their 2-digit number by the 1-digit number and says the product aloud. The player with the greater product takes all six cards.

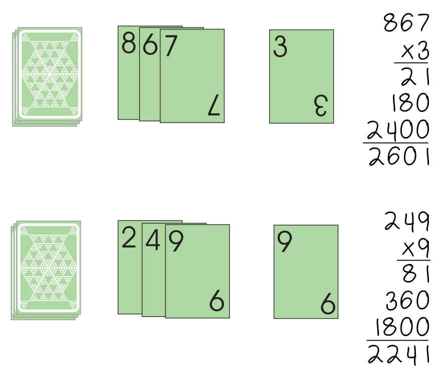
> *NOTE: The players may want to add a rule that the multiplicand, the first number, cannot start with a zero. If that rule is in play, switch the order of the cards.*



First round with both 2-digit numbers times 1-digit numbers calculated.

Wars occur when the products are equal, although this will rarely happen. If it does occur, each player places three extra cards face down and turns over three more cards that are multiplied together, as before. The player with the greater product takes all the cards.

During the second round, both players take four cards from their stack, making a 3-digit number multiplied by a 1-digit number. The player with the larger product takes all eight cards.



Second round with both 3-digit numbers times 1-digit numbers calculated.

The third round will use five cards from the stack to make a 4-digit number times a 1-digit number equation, with the winner taking all ten cards.

Continue playing, each player using three cards, then four, then five cards per round. The object of the game is to capture all the cards, which could take a while. For a shorter game, the winner is the player who has the most cards when the initial stacks are exhausted.

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DAY 17 – Traditional Multiplying

Needed materials. Paper and pencil or dry erase board and marker, Cotter Abacus, and Basic Number card deck

Multiplying with partial products. Tell the student to use their paper and pencil or dry erase board and marker and abacus to multiply 18 × 3 using partial products. [54] The solution is shown on the right. Keep this equation handy for future comparisons.

18
× 3
24
30
54

Multiplying traditionally. Explain: Writing all the partial products, like you have been doing, can be a lot of work. Today, you will learn to write just the answer, the product. To the right of the partial product calculation, write:

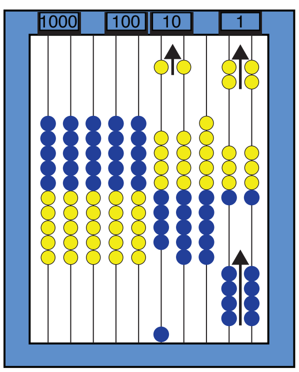
18
× 3

Tell them the multiplication is the same, but what you write down is different. Start by moving all the beads to the middle of the abacus and enter 18 at the bottom. Next, tell the student to multiply 8 × 3 by moving the 8 up and entering 24 at the top. See the first figure below.

Ask: How many beads are in the ones place? [4] Write 4 below the line. Ask: What other beads do we have? [2 tens] Write 2 **above the tens** in 18, as shown next to the first figure below.

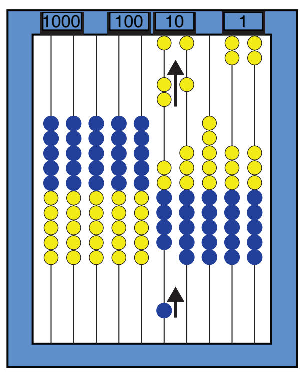
> *NOTE: The student needs to write down each action immediately after it occurs on the abacus. This allows the abacus procedure to parallel the traditional paper and pencil algorithm.*

Say: Let's multiply the tens next. Move the 10 up from the bottom, multiply 10 × 3, and enter 30 at the top. See the second figure below. Ask: How many tens are there altogether? [50; 20 from the carry plus 30] Tell the student to write 5 below the line in the tens place. Ask: What is the product of 18 × 3? [54] See the last figure.



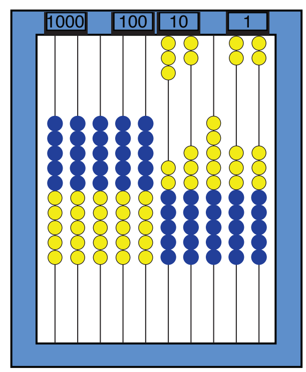
2
18
× 3
4

18 entered, then 8 × 3 = 24



2
18
× 3
54

10 × 3 + 20 (from carry) = 50



2
18
× 3
54

18 × 3 = 54

Multiplying by adding. Tell the student to multiply 18 by 3 by adding 18 + 18 + 18 in a vertical format next to the other calculations. Make sure they record the carries, the little numbers.

18
× 3
24
30
54

2
18
× 3
54

2
18
18
+ 18
54

Comparison. Ask: What do you notice about the three problems? [different ways to get the same answer] Where is the 2 of the 24 in all three equations? [under the first line of the partial products, above the 1 in 18, in the traditional and addition formats] Tell them to circle the 2 in each problem. Say: Where the 2 is written depends on the method used, and it is always in the tens place!

18
× 3
(2)4
30
54

(2)
18
× 3
54

(2)
18
18
+ 18
54

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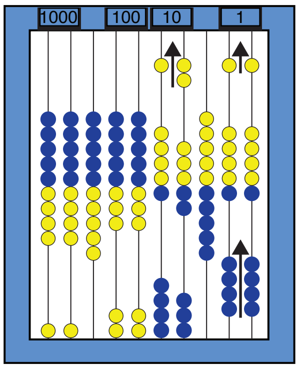
Another comparison. Tell the student they will solve 2478 × 4 three different ways: using partial products, traditionally written multiplication, and vertical addition. See the figures on the right.

$$\begin{array}{r} 2478 \\ \times\quad 4 \\ \hline 32 \\ 280 \\ 1600 \\ \underline{8000} \\ 9912 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\,3\,3} \\ 2478 \\ \times\quad 4 \\ \hline 9912 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\,3\,3} \\ 2478 \\ 2478 \\ 2478 \\ +\ 2478 \\ \hline 9912 \end{array}$$

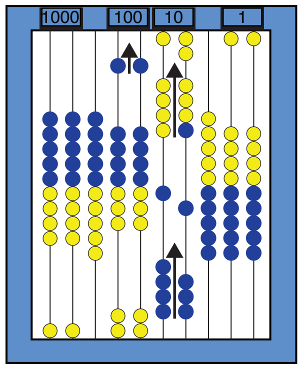
The abacus steps for the traditionally written method are shown below. If needed, guide the student through the process.

First, multiply 8 by 4; write 2 in the ones place and 3 above the tens. See the first figure below. Next, multiply 70 by 4, and then add it to the 3 tens already on the abacus, making 310, as shown in the second figure. Make the trade and write 1 in the tens place below the line and 3 above the hundreds, as shown in the third figure.



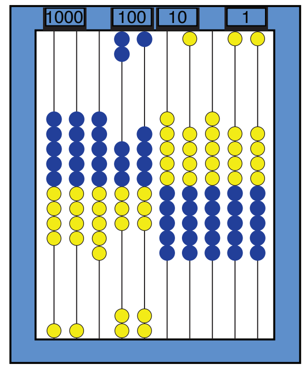
$$\begin{array}{r} {\scriptstyle \mathbf{3}} \\ 247\mathbf{8} \\ \times\quad \mathbf{4} \\ \hline \mathbf{2} \end{array}$$

2478 entered, then 8 × 4 = 32



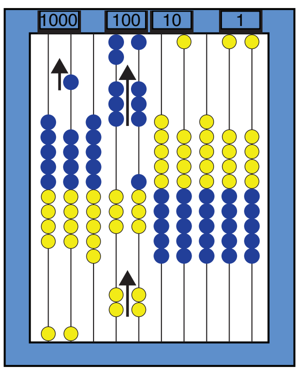
$$\begin{array}{r} {\scriptstyle 3} \\ 24\mathbf{7}8 \\ \times\quad \mathbf{4} \\ \hline 2 \end{array}$$

70 × 4 + 30 = 310



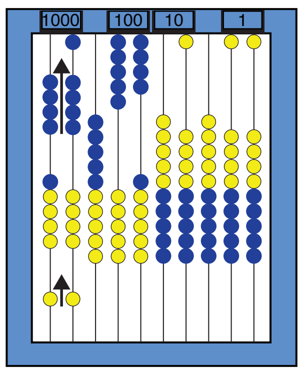
$$\begin{array}{r} {\scriptstyle \mathbf{3}\,3} \\ 2478 \\ \times\quad 4 \\ \hline \mathbf{1}2 \end{array}$$

After trading



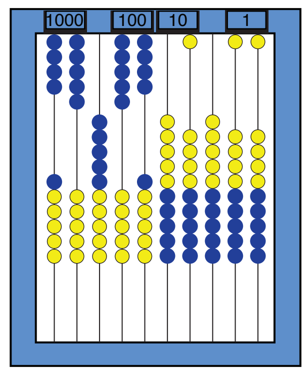
$$\begin{array}{r} {\scriptstyle \mathbf{1}\,3\,3} \\ 2\mathbf{4}78 \\ \times\quad \mathbf{4} \\ \hline \mathbf{9}12 \end{array}$$

400 × 4 + 300 = 1900



$$\begin{array}{r} {\scriptstyle 1\,3\,3} \\ \mathbf{2}478 \\ \times\quad \mathbf{4} \\ \hline \mathbf{9}912 \end{array}$$

2000 × 4 + 1000 = 9000



$$\begin{array}{r} {\scriptstyle 1\,3\,3} \\ 2478 \\ \times\quad 4 \\ \hline 9912 \end{array}$$

Final product

Then, 400 × 4 + 300 is 1900; write 9 in the hundreds place below the line and 1 above the thousands, as shown in the first figure above. Lastly, 2000 × 4 + 1000 is 9000, with the 9 written in the thousands place, as shown in the second figure above. The product of 2478 × 4 is 9912.

Say: You can see the carries, the 1, 3, and 3, are the same in the traditionally written and addition equations. Ask: Where is the carry of 3 hundred in the equation using partial products? [the 2 hundred is written with the additional 1 hundred from the trade of 30 + 80] See the figures on the right.

$$\begin{array}{r} 2478 \\ \times\quad 4 \\ \hline 32 \\ 280 \\ 1600 \\ \underline{8000} \\ 9912 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\,3\,3} \\ 2478 \\ \times\quad 4 \\ \hline 9912 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\,3\,3} \\ 2478 \\ 2478 \\ 2478 \\ +\ 2478 \\ \hline 9912 \end{array}$$

Turbo Multiplication War

Play this game from Day 14, but use the traditionally written method to calculate the products. The Cotter Abacus may be used as needed.

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DAY 22 – Multiplying by Three Digits

Needed materials. Paper and pencil or dry erase board and marker, Basic Number card deck, and Cotter Abacus, if needed

> *NOTE: Many curricula do not address multiplying by three digits. This topic is also not covered in most state standards. We have included multiplying by three digits because it increases understanding, shows the pattern of the process, and builds confidence.*

Review. Tell the student to write these three problems and solve. See the figures below.

```
                                  1 3
   2 5          1 3               2 5
   427          427               427
 ×   8        ×  50             ×  58
  3416        21350              3416
                                21350
                               24,766
```

Ask: How are the three problems related? [the first two multipliers add up to the third multiplier because 8 + 50 = 58] Did you annex a zero before multiplying by the 5 in the 50? [yes]

Multiplying by three digits. Tell the student to write the four problems shown below and solve the first two.

```
  1 1         1 1         1 1
  486         486         486         486
×   2       ×  20       × 200       × 222
  972        9720       97200
```

Ask the student how they might solve the third problem. Allow them time to think. If needed, guide them by asking: When multiplying by a multiple of ten, you annexed one zero; when multiplying by a multiple of a hundred, how many zeros need to be annexed? [2] Tell them to annex two zeros, then proceed to multiply 486 by the 2 of the 200. See the third figure above.

Ask: How are these four problems related? [the first three multipliers will add up to the fourth multiplier because 2 + 20 + 200 = 222] Will 486 × 2 plus 486 × 20 plus 486 × 200 equal 486 × 222? [yes] Tell the student to solve the fourth problem shown above. The solution is on the right.

```
    1 1
    1 1
    1 1
    486
  × 222
    972
   9720
  97200
107,892
```

Say: Multiplying by multiples of a hundred, like 200 or 500, is similar to multiplying by multiples of ten, like 30 or 80: annex the needed zeros, then multiply.

Another problem. Tell the student to write the problem 549 × 387, as shown in the first figure below. Then, tell them to multiply 549 × 7. See the second figure below.

```
                                        1 2
                           3 7          3 7
              3 6          3 6          3 6
  549         549          549          549
× 387       × 387        × 387        × 387
             3843         3843         3843
                         43920        43920
                                     164700
                                    212,463
```

Tell them to continue by multiplying 549 × 80, then finish by multiplying 549 × 300 and find the product. See the third and fourth figures above.

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A third problem. Tell the student to write the problem 439 × 728, as shown in the first figure below. Ask: Do you think it matters that the multiplier, the bottom number, is greater than the multiplicand, the first number? [answers will vary] When considering 4 × 7, does it matter that 7 is more than 4? [no] So will it matter that 728 is more than 439? [no]

Tell the student to find the product. See the figures below.

			2 6
		1	1
	3 7	3 7	3 7
439	439	439	439
× 728	× 728	× 728	× 728
	3512	3512	3512
		8780	8780
			307300
			319,592

Discussion. Ask: Is the process any different when multiplying a number by a 3-digit number compared to a 2-digit number? [no, other than the number of zeros to annex]

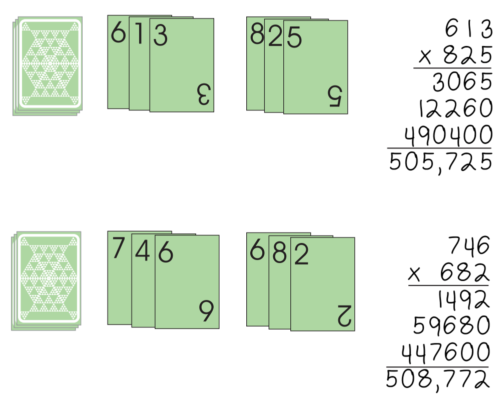
Turbo Multiplication War—Supreme Level

This game is yet another variation of the Turbo Multiplication War game, found on Day 14. Using slightly more than half of the basic number cards, give half of the cards to each player. Players will also need paper and pencil or dry erase board and marker. Some players may want to use an abacus.

Both players take the top six cards from their stacks, lay them down face up, and slide the first three cards together to create a 3-digit number. Then, layer the next three cards together to create another 3-digit number; this will be the multiplier. Each player multiplies their numbers together and says the product aloud. The player with the greater product takes all 12 cards.

During the second round, both players take seven cards from their stack to make a 4-digit number and a 3-digit number. The players multiply them together, and the winner takes all 14 cards.

Continue playing with each player using six cards, then seven cards per round. The object of the game is to capture all the cards. For a shorter game, the winner is the player who has the most cards when the initial stacks are exhausted.



First round with 3-digit numbers times 3-digit numbers.

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DAY 28 – Dividing 4-Digit Numbers on the Cotter Abacus

Needed materials. Paper and pencil or dry erase board and marker, Cotter Abacus, Short Multiplication table (Appendix p. 1) if needed, Multiplication card deck, and Basic Number card deck

Review. Ask the student to write and solve 38 ÷ 5 using the division house. Tell them to use the Cotter Abacus or Short Multiplication table as needed. [7 r3] See the figure below. Make sure the 7 aligns with the 8 because they are both in the ones place.

$$\begin{array}{r}7\ \text{r3}\\ 5\overline{)38}\end{array}$$

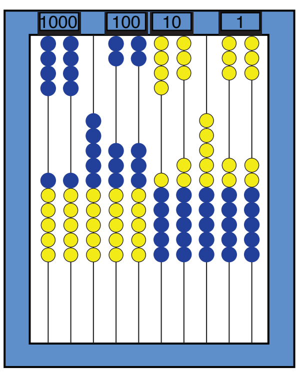
Tell the student to check their work by multiplying the quotient by the divisor, then adding the remainder. [7 × 5 + 3 = 38]

Continue with 28 ÷ 6 [4 r4] and 71 ÷ 8, [8 r7] checking their own work. Calculate other equations until they are comfortable writing division problems with the division house and solving them.

Dividing by 2 on the abacus. Because it often works better for a student to first watch a demonstration, rather than copy each step, tell the student to watch while you perform 4-digit division on the abacus.

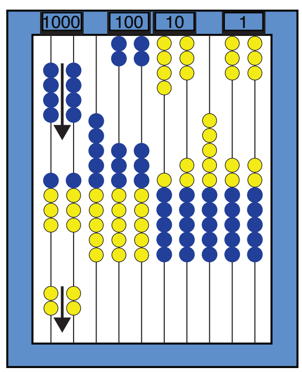
Write: $\begin{array}{r}\_\,\_\,\_\,\_\\ 2\overline{)8476}\end{array}$. Include the little lines above the dividend. Tell the student the little lines will help them know where to write each digit of the quotient.

Say: We will divide 8476 by 2 on side 2 of the abacus. Move all the beads to the middle of the abacus. Then, enter 8476 at the top, as shown in the first figure below. Explain that the answer will be entered at the bottom of the abacus, which is opposite of the process of multiplication.



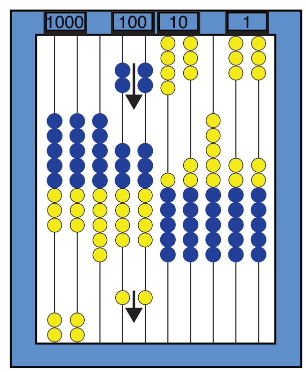
$$\begin{array}{r}\_\,\_\,\_\,\_\\ 2\overline{)8\,4\,7\,6}\end{array}$$

8476 entered



$$\begin{array}{r}\mathbf{4}\,\_\,\_\,\_\\ 2\overline{)\mathbf{8}\,4\,7\,6}\end{array}$$

8000 ÷ 2 = 4000



$$\begin{array}{r}4\,\mathbf{2}\,\_\,\_\\ 2\overline{)8\,\mathbf{4}\,7\,6}\end{array}$$

400 ÷ 2 = 200

Say: Watch while we divide 8 thousand by 2 on the abacus. We will do this by moving the 8 thousand from the top and entering the quotient, 4 thousand, at the bottom. Write the 4 in the thousands place. Ask: What is 8000 ÷ 2? [4000] See the second figure above.

Say: Next, we divide 4 hundred by 2. Ask: What is 400 ÷ 2? [200] Move 4 hundred from the top and enter 2 hundred at the bottom. Ask: What do we write? [2 in the hundreds place] See the third figure above.

Say: Now, look at the 7 beads in the tens column: 7 ÷ 2 is 3 r1. To show this, we will move 6 beads from the top, leaving 1 ten, and enter 3 tens at the bottom. Ask: What do we write? [3 in the tens place] See the first figure on the next page.

Ask: What is the value of the beads at the top of the abacus now? [16] Say: To show 16 ones, we write a small 1 before the ones place in the dividend. See the first figure on the next page. Ask: What is 16 divided by 2? [8] Move the 16 from the top and enter 8 at the bottom. Ask: What do we write? [8 in the ones place]

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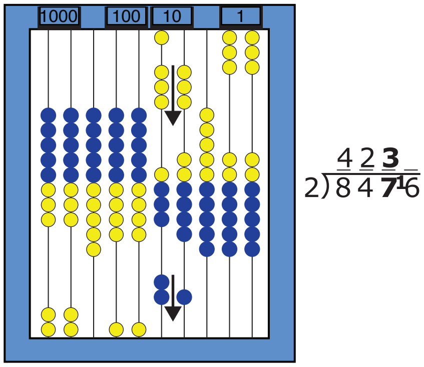
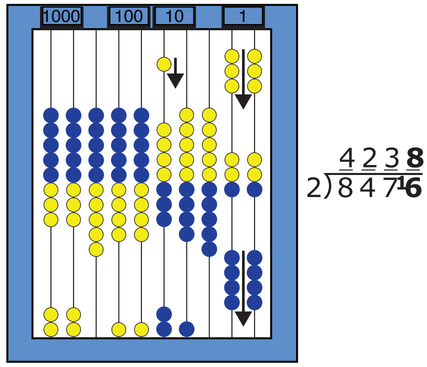
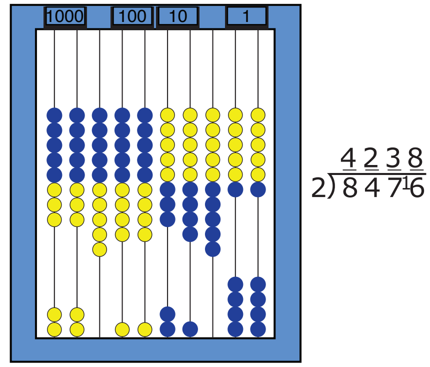


60 ÷ 2 = 30



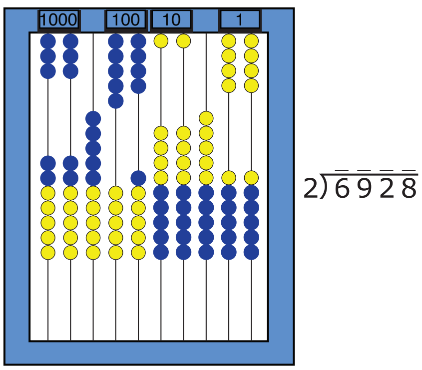
16 ÷ 2 = 8



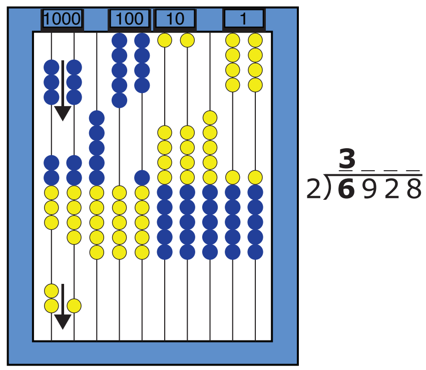
8476 ÷ 2 = 4238

Ask: What is 8476 ÷ 2? [4238] See the last figure above.

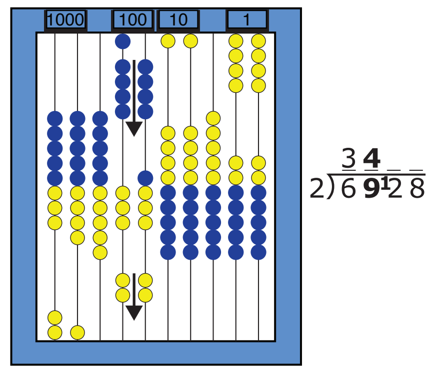
Another problem. Tell the student to calculate 2)6928 on the abacus, recording each step as they progress to the solution. Some students may need to use the Short Multiplication table. Give them time to work. See the figures below.



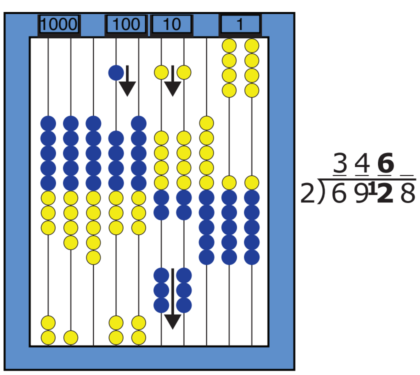
6928 entered



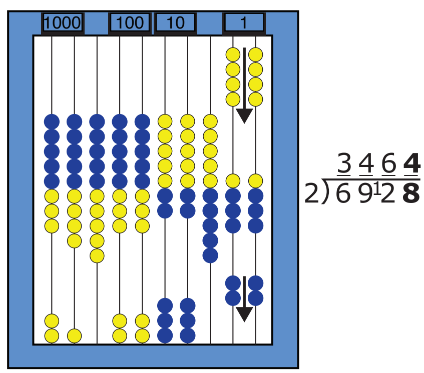
6000 ÷ 2 = 3000



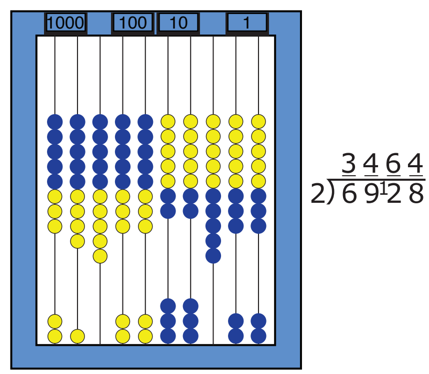
900 ÷ 2 = 400 with 100 remaining



120 ÷ 2 = 60



8 ÷ 2 = 4

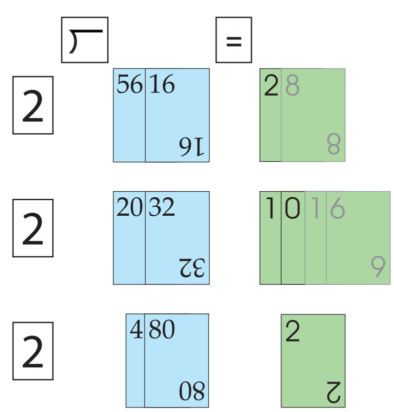


6928 ÷ 2 = 3464

Quotient, No Remainder

This 2- to 4-player game is a version of Quotient and Remainder, found on Day 26. Use the **even-numbered** multiplication cards, half the basic number cards, and all the 0s, 1s, and 2s. Notes with "⟌," "=," and several "2" notes will also be used. Use the abacus.

Two multiplication cards will form the dividend. The "2" notes will be used as the divisor for **each** row. A player plays as many cards as possible to form the quotient. Recycle the basic number cards, as needed. The player who collects the most multiplication cards wins.



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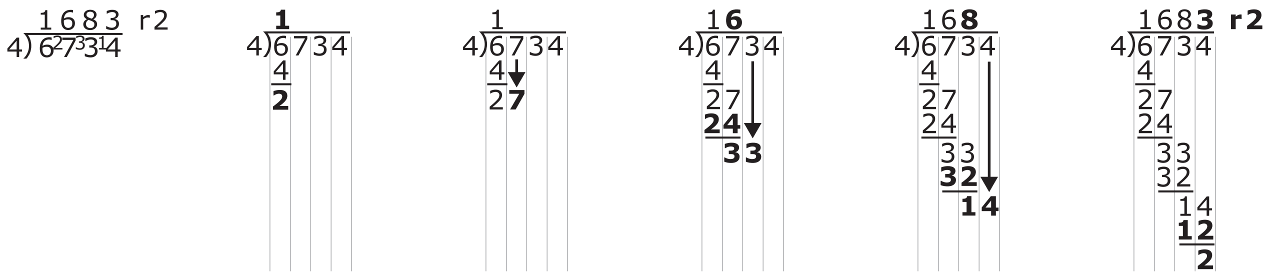
DAY 38 – Short Division to Long Division

Needed materials. Paper and pencil or dry erase board and marker, lined paper or **copy** of Grid for Long Division (Appendix p. 14), Multiplication card deck, and Basic Number card deck

Comparing 1-digit short division to long division. Tell the student today we are going to show how short division is linked to long division. Say: Let's start with a simple short division problem. Solve 6734 ÷ 4. Be certain to write the little numbers, even if you are able to do them mentally. See the first figure below.



Tell the student to write the problem again using the grid paper provided in the appendix or draw vertical lines between each number of the dividend, as shown above.

Say: Looking at the 6 in 6000, divide 6 by 4. Write the 1 above the division house, just like you did with short division. Instead of writing the remainder, 2, as a little number, we will be writing down the steps you did mentally.

Ask: What is 4 × 1? [4] Say: Write 4 below the 6 in the dividend, keeping inside the vertical lines. Next, subtract the two vertical numbers, recording the answer below: 6 – 4 = 2. See the second figure above.

NOTE: Although subtraction is being done, the subtraction symbol is not generally written during the calculations. If the student wants or needs to record the subtraction sign, that is acceptable.

Say: Now, "bring down" and rewrite the 7 (from 700 in the dividend) next to the 2. See the third figure. Ask: Looking at the 27 you just wrote, where do you see that number in the short division problem? [the little 2 next to the 7 of the dividend] Say: The process we just wrote, 4 × 1 and 6 – 4 with a remainder of 2, is all done mentally with short division. It might seem silly to write all of this when it is so much easier to calculate mentally, but this helps us see how long division actually works.

Tell the student to continue dividing. Ask: What is 27 ÷ 4? [6 r3] Tell them to write the 6 above the 7 of the dividend, multiply 4 × 6, write the product below the 27, subtract 27 – 24, and record the 3. Bring down the next digit, 3, to create 33. See the fourth figure above. Have the student identify where 33 is found in the short division problem.

Ask: What is 33 ÷ 4? [8 r1] Tell them to record the 8 above the division house, calculate 4 × 8, record the product, subtract 33 – 32, and record the difference, 1. Finally, bring down the 4 from the dividend. See the fifth figure above. Again, ask the student to identify where the 14 is found in the short division calculation. [little 1 written before the 4 of 6734] Tell them to finish the problem, as shown in the last figure above.

NOTE: Some students may wonder why we "bring down" the numbers from the dividend. We do this to give more room to write the remainder, then "bring down" the next place value segment and continue dividing. In other words, it's the extended version of writing the little numbers between the digits of the dividend.

Ask: Which way is easier, short division or long division? [short division] Say: As we have learned, short division is easy for 1-digit divisors. And we know how to use double short division for 2-digit divisors that have factors. Since short division will not work for 2-digit divisors that do not have factors we can use, there is a use for long division.

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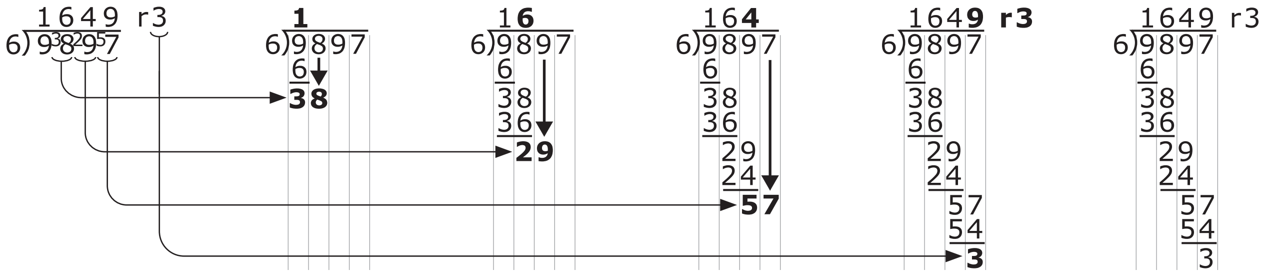
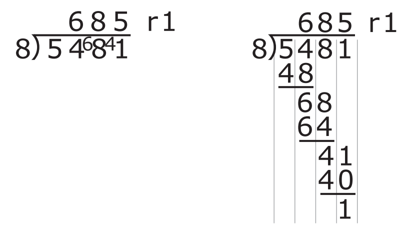
Another problem. Tell the student to solve 9897 ÷ 6 using short division, then solve again using long division. See the figures below.

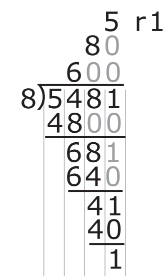


Ask the student to point out where the 38, 29, and 57 in the long division problem are found in the short division problem. See the lines drawn above.



One more problem. Tell the student to compare one more short division calculation with the long division calculation: 5481 ÷ 8.

Again, make sure they do the short division first so the alignment between short division and the long division steps is clear. If needed, ask them to point out where the 68 and 41 are in both equations. See the figures on the right.



NOTE: If the student needs additional understanding with this process, consider the figure on the right. It is similar to multi-digit multiplication using partial products, as discussed on Days 11, 12, and 13. This expanded form could be called "longer division."

Notice how the 600 and 80 are stacked in the quotient rather than just writing the 6 and 8 as done in long division. Also, notice how the numbers below the dividend record the zeros that are only assumed in the traditional format. This illustrates the "bring down" portion of the process!

Summary. Ask: Does it surprise you that short division and long division are so similar? [answers will vary] If appropriate, discuss the student's prior fears and concerns with long division.

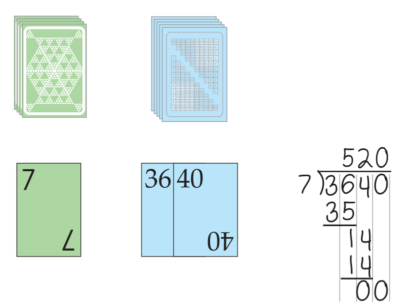
Long Division Game—Level 1

This game is similar to Quotient and Remainder—Next Level from Day 29, but rather than forming the results with cards, the student will write each step as they find the solution. It can be played as a solitaire game or as a multiplayer game. Every player will need something to write on. Use a copy of the grid paper from Appendix p. 14, lined paper (turned so the lines run vertically), or draw vertical lines for each problem.

If two or more players are playing, this game creates an opportunity for everyone to work together and check their work with each other as they progress through the problem. This game is not a race but rather a time to collaborate and help each other.

The multiplication cards, without the 100-card, and half of the basic number cards with no 0s, 1s, or 10s will be needed. Turn over two multiplication cards from the stock. Overlap one card with the other to create a 3- or 4-digit dividend. Next, turn over a basic number card from the stock to become the divisor.

Each player solves the equation using long division. If they prefer to use short division first, they may do that. Play at least four hands.



A zero is in the ones place of the dividend. The zero is "brought down" and recorded in the quotient. The double 0s are not necessary but help some students.

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DAY 44 – Skills Day

Needed materials. Day 44 Problems (Appendix pp. 15a and 15b), paper and pencil or dry erase board and marker, lined paper or copy of the grid (Appendix p. 14), and Multiplication card deck

Review. Say: We have learned two ways to handle 2-digit divisors: double short division and long division. Ask: Can double short division be used for all 2-digit divisors? [no] What 2-digit numbers will it work for? [numbers with single-digit factors, other than 1] If you can't use double short division, what can you use? [long division]

Tell the student that today we are going to read and work through some problems.

NOTE: For students who did not do Days 40, 41, and 42, encourage them to write the multiples of the divisor when needed.

Problem 1. Give the student the Day 44 Problems and read the first problem out loud:

A large neighborhood group of 32 families decided to have a garage sale. A garage sale, sometimes called a yard sale, is the selling of secondhand items from someone's garage or yard for a bargain price, sometimes as low as 10¢. Items for sale may include kitchen tools, clothing, sporting goods, books, and toys.

The group decided that keeping track of everyone's items and sale prices was too much trouble, so they agreed to split the total sales equally. At the end of the event, their total sales were $9856. How much money did each family make at the garage sale?

Read the problem again. Give them time to think about and solve the problem. If needed, ask: What question needs to be answered? [what was each family's share of the sales] How many families were involved? [32] How much was the total sales? [$9856] What is the equation to solve? [9856 ÷ 32] Tell them to solve the equation. [$308 to each family] See three possible ways to solve 9856 ÷ 32.

```
   308          308
4)1232      32)9856
8)9856         96
                25
                 0
   308          256
8)2464          256
4)9856            0
```

Problem 2. Read the next problem out loud:

Wait! When the families agreed to split the total sales equally, there was a condition that any money made from larger-priced items would go directly to the selling families, and the adjusted total would be split equally.

The Marsh family sold their treadmill for $200, the Yang family sold two couches for $70 each, the Ramirez family sold their old TV for $120, and the Levi family is moving into a condo and sold their garden tools and lawn mower for $500. What is the adjusted total that each family receives?

Read the problem again. Give them time to think about the problem and solve it. If needed, ask: What was the total sales from the event? [$9856] How much needs to be taken out of the total before it is divided equally? [$960; $200 + $70 + $70 + $120 + $500] Make sure the student notices and includes the Yang's two couches for $70 each.

Ask: What is the new sales total? [$8896; 9856 – 960] How do you find the new equal share? [divide 8896 by 32] See possible ways to solve the equation on the right. Ask: What is the new adjusted share? [$278]

```
   278          278
4)1112      32)8896
8)8896         64
               249
               224
   278          256
8)2224          256
4)8896            0
```

Ask: Does each family get exactly $278? [no, because the Marsh, Yang, Ramirez, and Levi families get additional amounts from the sales of their larger-priced items] So, what does the Levi family get from the garage sale? [$778; 278 + 500] What did the Yang family get from their sales? [$418; 278 + 140] How about the Marsh family? [$478; 278 + 200] And the Ramirez family? [$398; 278 + 120]

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Problem 3. Read the problem out loud:

> Some of the families' kids asked if they could make cookies to sell at the garage sale. Seventeen kids were interested in participating. The parents decided to donate the supplies for the baked goods, and the kids kept their sales separate from the garage sale income.
>
> Each kid made 60 cookies and then got together to package 13 cookies per plate. They decided some packages would have one type of cookie, and other packages would have a variety of cookies. How many packages of cookies could they make?

Read the problem again. Give them time to think about the problem and solve it. If needed, ask: What question is being asked? [how many packages of cookies can be made] How many cookies are in each package? [13] How many cookies are there altogether? [1020] If a student is unsure of the total number of cookies, ask: How many kids were bringing cookies? [17] How many cookies did each kid bring? [60] So, how many cookies are there altogether? [1020; 17 × 60] See the first figure on the right.

```
  17
× 60
1020
```

Continue by asking: So, how many packages of cookies can they make? [78 because 1020 ÷ 13 = 78 r6] See the second figure on the right.

```
      78 r6
13)1020
    91
    110
    104
      6
```

If the student replies that the answer is 78 r6 rather than 78 plates of cookies, discuss what a "remainder 6 plate" might look like. Continue to discuss what they could do with the extra six cookies. Possibilities include eating the extra cookies, making a smaller package of cookies, adding one extra cookie to plates with smaller cookies, or using the leftover cookies for samples.

Problem 4. Read the last problem out loud:

> At the end of the event, the kids sold all their packages of baked goods, all at the same price, and raised $858. What did they sell each package of cookies for? How much does each kid receive from the sale of their baked goods?

Read the problem again. Give them time to think about the problem and solve it. If needed, ask: What is the first question being asked? [how much did they sell each package of cookies for] How much were the total sales? [$858] How many packages were sold? [78, from Problem 3] Did the price vary for any of the cookie packages? [no] So, what was the price for each package? [$11; 858 ÷ 78] See the figure on the right.

```
     11
78)858
   78
    78
    78
     0
```

Continue to ask: What is the second question being asked? [how much does each kid receive from the sales] Give the student time to think. If needed, ask: How many kids were involved in this sale? [17] What were the total sales? [$858] So, what does each kid get for their work? [$50; 858 ÷ 17 = 50 r8] See the second figure on the right.

```
     50 r8
17)858
   85
    08
     0
     8
```

NOTE: If the student calculates the answer as $50.47, that is accurate; however, we are not focusing on decimals in these lessons.

Again, check that the student does not provide the answer 50 r8. If they do, discuss how the kids would spend "50 r8" dollars. Ask: What does the "r8" mean? [that $8 is left over from the equal split] What should the kids do with the remaining $8? [answers will vary]

Game: Multivide Game—Boss Level

NOTE: If there is time and interest, play this game. Instructions are found on Day 43.

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