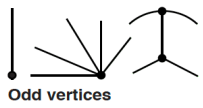
Most recent update: August 15, 2025

RightStart™ Mathematics

Corrections and Updates for Grade 7 Lessons and Worksheets, second edition

LESSON/WORKSHEET/SOLUTIONS			CHANGE DATE	CORRECTION OR UPDATE
Objectives			10/24/2023	Objectives were added to the Lesson book. See attached **PDF**.
		Solutions 9-2	10/14/2020	The answer for Problem 11 should be r = **2.8** in, not 2.75, and $A = \pi \times$ **2.8**, not A = π × 2.75, giving an answer of **24.6** in^2, not 23.8 in^2. If the actual value for the radius found in the first step, 2.753380515, is used to calculate the area, the area will be 23.81674146 in^2, or 23.8 in^2 rounded.
	Worksheet 19	Solutions 19	12/27/2019	The image for Problem 4 has been changed. See attached **PDFs**.
	Worksheet 21		12/27/2019	Two plaids are outlined in the new worksheet. Instructions are changed slightly. See attached **PDF**.
	Worksheet 22-2		12/27/2019	For Problem 14, the instructions are changed to "Draw a **regular** tessellation…" rather than "Draw a pure tessellation…"
		Solutions 36-1	12/27/2019	In the notes for the remaining six products, the final problem of 45 × 12 can have the following options: 15 × 36, 30 × 18, **20 × 27**, and **54 × 10**. The solution of 60 × 9 is an error.
		Solutions 38-2	06/27/2023	Problem 14 solution, third line of the calculations, should read 255 = **144** + u^2, not 255 = 1442^2 + u^2.
	Worksheet 39-3	Solutions 39-1	12/27/2019	For Problem 25, the instructions are changed to "Find p, t, and ***r***" and the value of r is removed from the graphics. See attached **PDF**. In the Solutions for Problem 25, when the calculated value for r is used in the third equation, it should be 5.4/**4.1**, not 5.4/4.0.
Lesson 41			08/15/2025	The older Safe-T Compass is being replaced by the newer Slide N' Measure Compass. See the instructions for both compasses with the attached **PDF**.
	Worksheet 41	Solutions 41	12/27/2019	Two final questions are asked. See attached **PDF**s.
Lesson 42	Worksheet 42-1	Solutions 42-1	08/15/2025	Worksheet has been modified for the new Slide N' Measure Compass. See attached **PDFs**.
Lesson 42	Worksheet 42-2	Solutions 42-2	08/15/2025	Worksheet has been modified for the new Slide N' Measure Compass. See attached **PDFs**.
	Worksheet 45	Solutions 45	08/15/2025	Worksheet has been modified for the new Slide N' Measure Compass. See attached **PDFs**.
	Worksheet 48-1		12/27/2019	Problem 5 has been updated with the size of the starting rectangle. See attached **PDF**.
		Solutions 48-2	12/27/2019	The first measurement for Problem 6 is **10.2**/6.3 = **1.6**, not 10.1/6.3 = 1.7
Lesson 57			06/03/2022	At the bottom of page 91, the first figure in the second set of graphic is changed to be a point and a single line as shown here.  Odd vertices
		Solutions 74-2	05/05/2021	In the calculations for Problem 28, S(X) should be 6 × **14**^{2}, not 6 × 142.
		Solutions 75-4	02/04/2020	The answer for Question 40 should be **A**, not B.

		Solutions 76-4	03/09/2020	The comment for Question 40 should say **600** cm^2, not 6 cm^2.
		Solutions 85-1	03/09/2020	The answer for Problem 14 needs convertion from centimeters to decimeters. It should be 63.63 cm **≈ 6.4 dm**.
	Worksheet 91-2	Solutions 91-2	03/11/2020	Question 3 should read "**If no faces are regular**, how many different ways will the package fit into the box?"
Lesson 95			03/18/2024	The picture in the lesson should be labeled "**These large tetrahedron skylights are at Washington DC's National Gallery of Art.**"
		Solutions 95-2	03/21/2023	Questions 20 and 21, the answers for Euler's theorem are switched around. The dodecahedron formula should be **12** + **20** = 30 + 2 and the icosahedron's formula should be **20** + **12** = 30 + 2.
		Solutions 101-2	04/13/2022	For Question 21, the solutions reference Q#18 when it should be **Q#19**. For Question 24, the solution should read empty = 100 – **61.7** = 38.3%, not 100 – 62 = 38.3%.
	Worksheet 107	Solutions 107	04/20/2020	The first ordered pair for equation 4 should read -4 + **12**, not -4 + 11, giving an answer of (**8**, 1), not (7, 1). The first ordered pair for equation14 should read -2 + **-6**, not -2 + -5, giving an answer of (**-8**, 1), not (-7, 1). Graphed image is correct.
Lesson 113			04/20/2020	In the first Extra, the beginning sentence should read "…would be 0.25 × *n* , or 0.25 × **5**, which is $1.25", not "…would be 0.25 × *n* , or 0.25 × 10, which is $1.25."
	Worksheet 113	Solutions 113	04/20/2020	Question 3 should read "…household using between five and **eight** units...", not ...between five and nine units..."
	Worksheet 114-2	Solutions 114-2	04/20/2020	Problem I (in the second column) *x* -value equation should be 12^2 – **141**, not 12^2 – 121. Answer of 3 is correct.
		Solutions 114-3	04/27/2020	The solutions for Problem 38 should be: $g^2 = 92 + 749$ $g = \sqrt{841}$ $g = 29$
	Worksheet 116	Solutions 116	04/27/2020	The second to last inequality is change to **3 – *e* > 1/2**, not 3 – *e* > *x* . Solution of *e* < 5/2 has not changed.
	Worksheet 117		04/27/2020	The wording for Questions 7 and 8 have changed to recognize that at 0ºC and 32ºF water can be both frozen or liquid and that at 100ºC and 212ºF water can be both liquid or gas. See attached **PDF**.
Lesson 119	Worksheet 119	Solutions 119	10/14/2020	Half way down the lesson page, the section Solving absolute value eqution has been changed as there was a significant error. See attached **PDF** for the new lesson. Problem 6 has been changed on the Worksheet and Solutions to **|*x*| = 5 – 4**, not |*x*| = 4 – 5. Answers are not changed and remain as *x* = 1 and *x* = -1.
		Solutions 120	05/18/2022	The answer for Question 6 should be **1.58**, not 1.08, as the increase for the first set of numbers.
		Solutions 122	05/06/2020	The answer for Question 8b should be **10%**, not 20%.
	Worksheet 123-2	Solutions 123-2	10/14/2020	Problem 13 has changed and the data for Questions 14 through 17 has been changed. See attached **PDFs**.
		Solutions 124-6	05/05/2021	The number of hexagons for the truncated octahedron is **8**, not 6.
	Worksheet 124-8	Solutions 124-8	05/11/2020	The data for Questions 92 through 96 has been changed. See attached **PDFs**.
	Worksheet 125-8	Solutions 125-8	05/11/2020	The data and questions for Questions 87 through 90 have been changed. See attached **PDFs**.

RIGHTSTART MATHEMATICS OBJECTIVES FOR GRADE 7

Name _____ Year _____

	Trimester 1	Trimester 2	Trimester 3
Numeration			
Understands the difference between rational and irrational numbers*	N/A	N/A	
Understands that subtraction of rational number is the same as adding negative number	N/A	N/A	
Understands and applies absolute value	N/A	N/A	
Understands and applies central numbers, i.e., arithmetic mean, median, and mode	N/A	N/A	
Solving Equations			
Solves problems involving rational numbers using the four operations			
Applies properties of operations, ie., associative, commutative, and distributive, as strategies to solve problems			
Solves problems using positive and negative rational numbers	N/A	N/A	
Converts rational numbers between fractions, decimals, percents, and whole numbers			
Solves multi-step algebraic equations using distributive property including rational number coefficients*	N/A		
Evaluates expressions and writes answers in expanded form or in scientific notation, using positive and negative exponents*	N/A	N/A	
Understands and applies square roots and cube roots to solve problems*			
Problem Solving			
Solves multi-step one-variable equations involving coefficients, exponents, and parentheses			
Persists and finds more than one way to solve problems			
Knows and applies the distance formula to solve real-life problems	N/A		
Solves geometric problems by measuring lengths and computing areas from a scale drawing			
Ratic			
Evaluates proportional relationships to determine equality			
Identifies and analyzes the constant relationship, unit rate, in an equation, table, or graph			
Explores the golden ratio phi, Ø, and sees examples in the real world	N/A		
Applies proportional relationships to real-life situations	N/A		
Computes unit rates associated with ratios	N/A	N/A	
Writes proportional relationship equations	N/A	N/A	
Understands and applies trigonometry ratio calculations, specifically sine, cosine, and tangent	N/A		
Coordinate System			
Understands and plots positive and negative numbers on a line or coordinate plane	N/A		
Creates images and translations on a coordinate plane			
Statistics and Probability			
Understands probability are between 0 and 1, the larger number indicating more likely an event will occur	N/A	N/A	
Understands and applies measures of center and geometric mean to draw informal conclusions	N/A	N/A	
Geometry			
Uses tools to draw geometric shapes with specific conditions given, including side length and angles			
Solves real-world and mathematical problems involving area, surface-area, and volume of shapes and solids			
Understands, finds, and explains planes of symmetry and antiprisms	N/A		
Knows and applies formulas for circumference and area of a circle			
Applies area and circumference of a circle to real-world situations			
Applies Pythagorean theorem to find side length in real-world and mathematical situations*			
Explores and applies Fibonacci sequence and understands how it relates to the Golden Ratio	N/A		
Examines and creates nets and their relationship to solids	N/A		
Study Skills			
Understands and can explain geometric and mathematical terms			
Explores historic and cultural influences in math			
Develops independent learning skills			
Understands the importance of using available resources for independent learning			

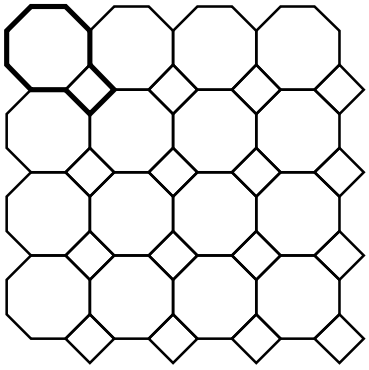
Name: _____

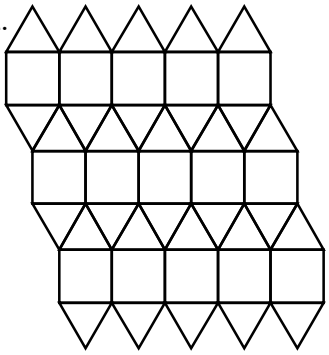
Date: _____

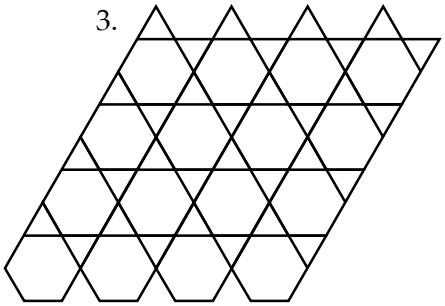
Find the basic unit for the three tessellations below. The first one is done for you.

Fill in the table to describe how to construct the tessellations by translating the basic unit. Use millimeters.

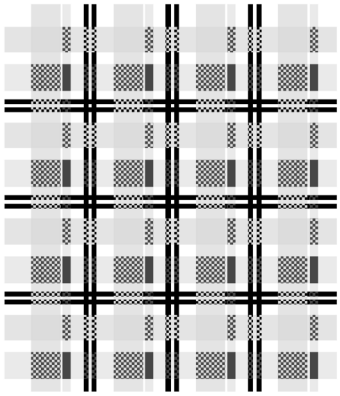
Problem	To make first row	To make next rows
1		
2		
3		

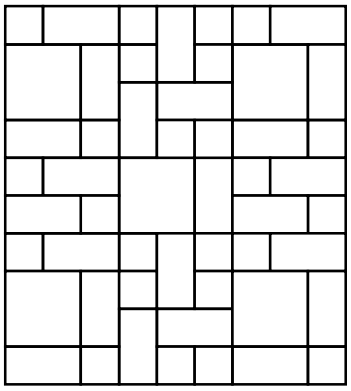
1. 

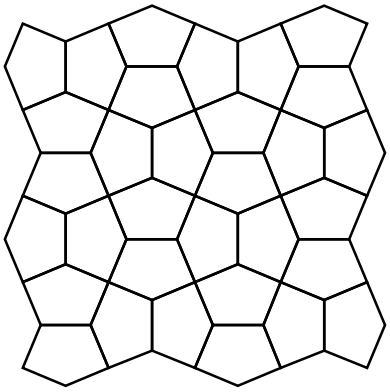
2. 

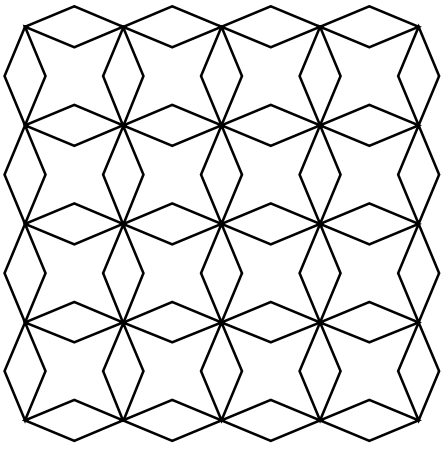
3. 

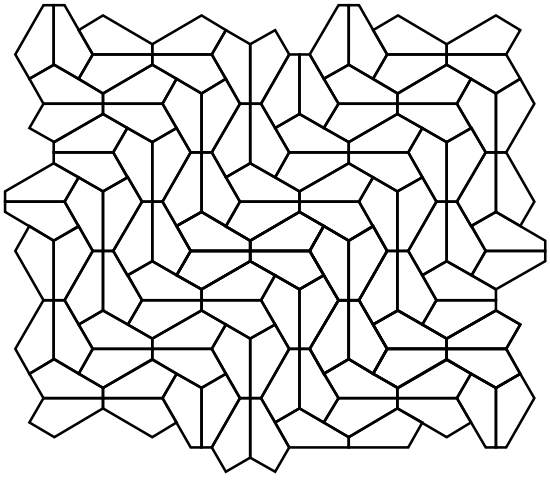
Find the basic unit.

4. 

5. 

6. 

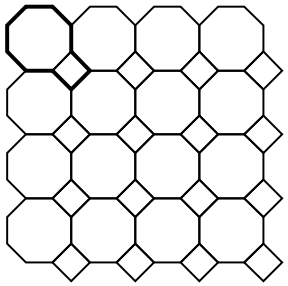
7. 

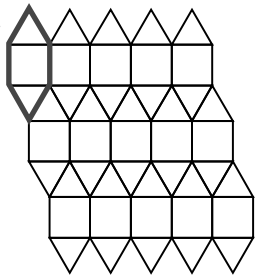
8. 

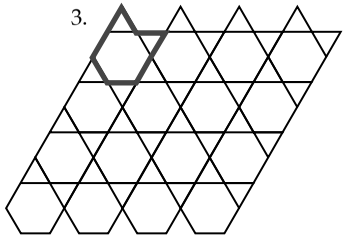
Find the basic unit for the three tessellations below. The first one is done for you.

Fill in the table to describe how to construct the tessellations by translating the basic unit. Use millimeters.

Problem	To make first row	To make next rows
1	(11, 0)	(0, -11)
2	(7, 0)	(3.5, -13)
3	(10, 0)	(-5, -8.6)

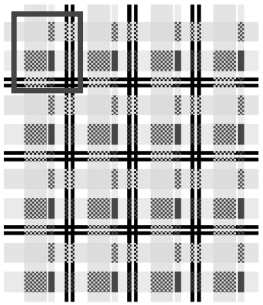
1. 

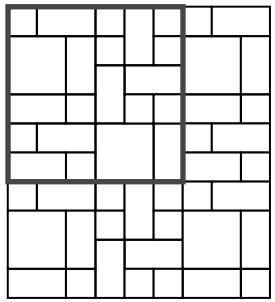
2. 

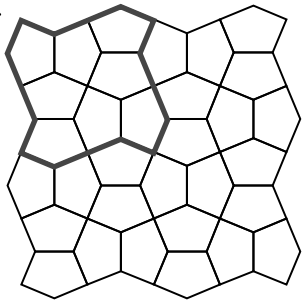
3. 

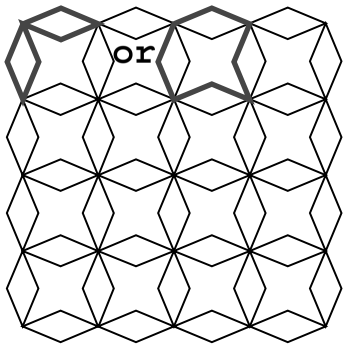
[THERE ARE OTHER SOLUTIONS.]

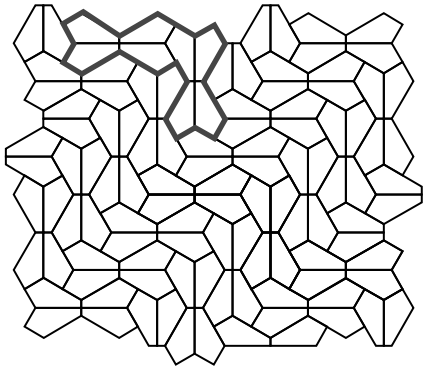
Find the basic unit.

4. 

5. 

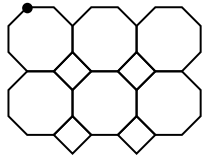
6. 

7. 

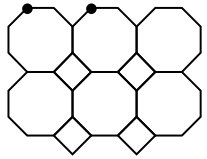
8. 

[THERE ARE OTHER BASIC UNITS.]

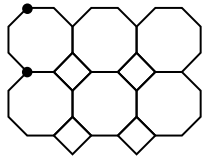
NOTES: If the student needs help finding the translations, start by picking a vertex on one unit. See the first figure on the right.



Then identify the corresponding vertex on the next unit. See the second figure. Measure the distance. In this situation, it is an 11 mm horizontal, x-value, move to the right, which is recorded as a positive 11. There is no vertical movement, so 0 is the y-value. The translation is (11, 0) with the x-value recorded first.



To find the next row, use the same procedure. Find the same vertex on a unit in the next row. See the third figure. Measure the distance; it will be 11 mm vertical and 0 mm horizontal. Because the distance vertically is down, or negative, the translation is written as (0, -11).



When finding the basic pattern, it will be the smallest unit. There are other options than what is shown here.

DICTIONARY TERMS: unit, pattern

Name: _____

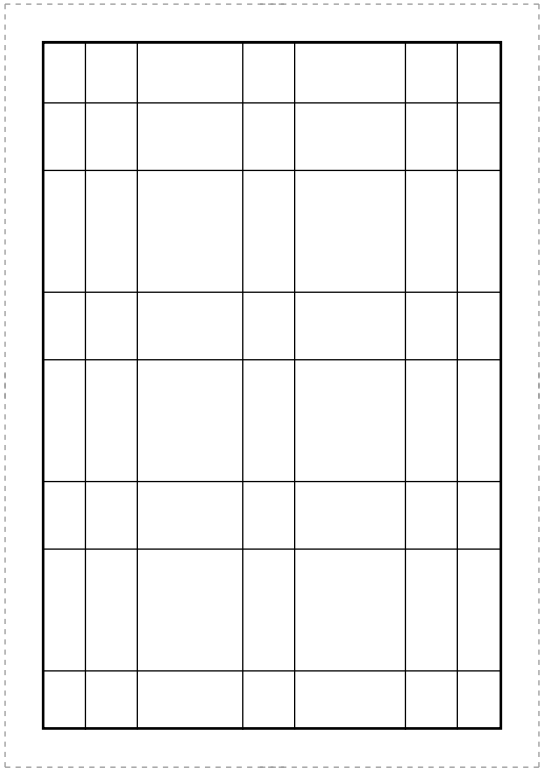
Date: _____

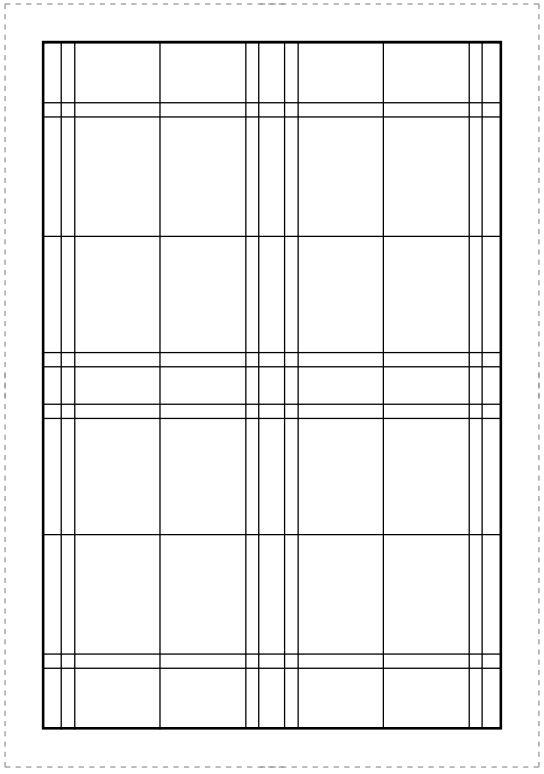
Worksheet 21, Tartan Plaids

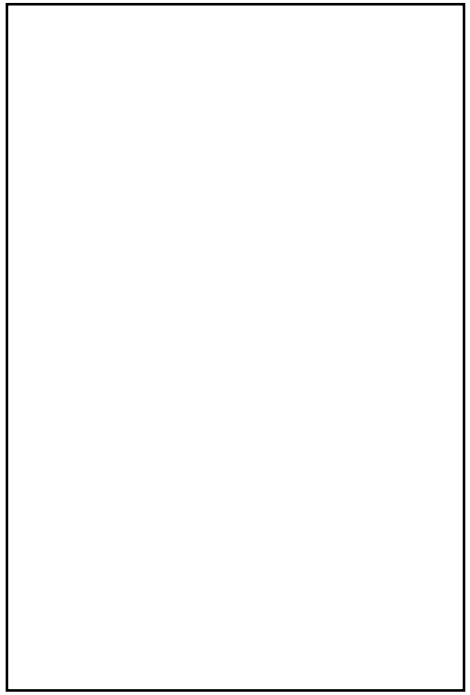
Fold and cut your tracing paper into three rectangles, each approximately the size of the dotted rectangles below.

For each rectangle, follow the instructions to design a plaid and color it.

1. Two plaids are outlined for you. Draw an outline for the third rectangle. Create a systematic design.
2. Center the tracing paper over a rectangle and tape the top in place.
3. On the tracing paper, systematically color each of the horizontal weft sections of the plaid all the way across.
4. Lift the tracing paper and color the vertical warp sections, all the way down.
5. Return the tracing paper to cover the rectangle and see each plaid design.







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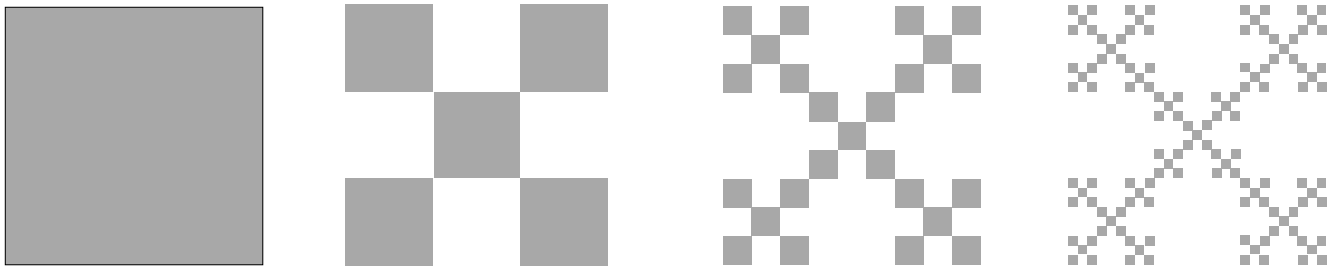


Iteration 0 Iteration 1 Iteration 2 Iteration 3

23. Using the Box Fractals above, complete the table below.

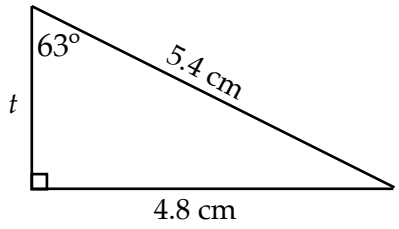
Iteration	**Number of Squares (use multiplication)**	**Number of Squares (use exponent & numeric form)**
0		$5^0 = 1$
1		
2		
3		
4		

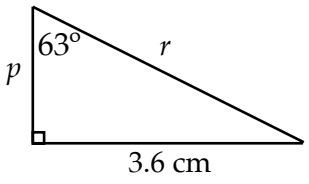
24. Solve these proportions. You may simplify the fraction before cross multiplying.

$\frac{p}{24} = \frac{5}{12}$ $\frac{7}{21} = \frac{r}{3}$ $\frac{1}{2} = \frac{e}{67}$

p = ____ r = ____ e = ____

25. Find p, t, and r. Round to the nearest tenth.





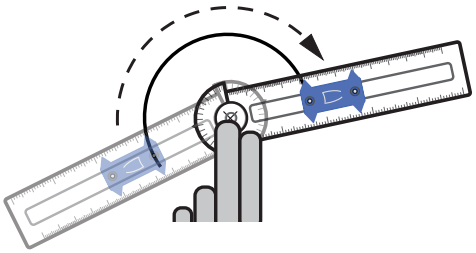
SLIDE N' MEASURE COMPASS

NEW effective August 2025; used in RightStart Mathematics Levels E, F, G, and H

** measures in inches and centimeters **

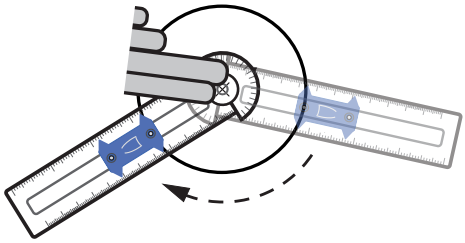
The Slide N' Measure Compass will draw circles with radii from 1.3 to 12 cm and 1/2 inch to 4-11/16 inches.

Align the center of the circle part of the compass with the center mark of your circle and hold it with your non-writing hand. Place the radius arm to the left. See the first figure on the right.



Position the slide to point to the desired radius. Put your pencil in the hole for that radius, then draw the circle in a clockwise direction. If you are left-handed, start the radius arm on the right side and draw the circle counterclockwise.

When your circle is almost complete, raise the heel of your hand, and continue drawing the circle until you have completed it. See the second figure.



To make more accurate circles, keep your pencil perpendicular to the paper. Also, keep light pressure against the outside of the hole while drawing.

A demonstration on using the Slide N' Measure Compass can be found at RightStartMath.com/geometry.

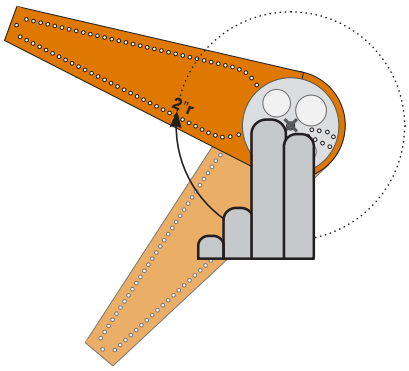
8/25

SAFE-T COMPASS®

used prior to September 2025 in Levels E and F

** measures in inches **

Start by aligning the center of the white rotator over the center of the circle being drawn and hold it with your non-writing hand. Find the hole marked with the desired radius measurement and insert the pencil.



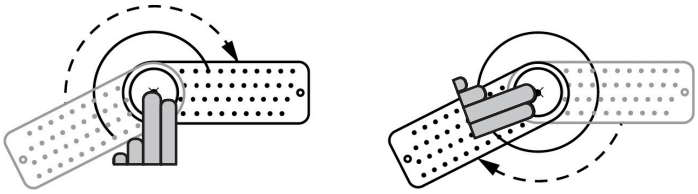
Keep the white rotator still and move the radius arm in an arc to draw the circle.

mmARC COMPASS

used prior to August 2025 in Levels G and H

** measures in millimeters **

Align the center of the movable part of the compass, the rotator, with the center of your circle and hold it with your non-writing hand. Place the radius arm to the left.



Put the pencil in the hole for the desired radius, then draw the circle in a clockwise direction. When the circle is almost complete, raise the heel of the hand. See the second figure above.

If you are left-handed, start on the right side and draw the circle counterclockwise.

To make more accurate circles, keep your pencil perpendicular to the paper. Also, press against the outside of the hole while drawing.

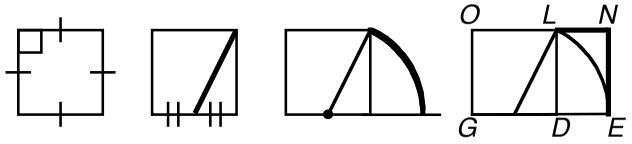
A demonstration on using this compass can be found at RightStartMath.com/geometry.

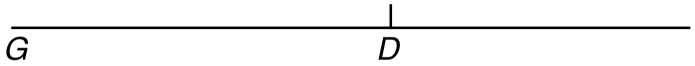
Name: _____

Date: _____

Worksheet 41, Golden Ratio

1. Construct a golden rectangle on the line below. Follow the instructions provided in the figures. Label the rectangles.





2–3. Measure and find the ratios of the longer side to the shorter side for both rectangles. Round to the nearest tenth.

$\frac{\overline{GE}}{\overline{OG}}$ =

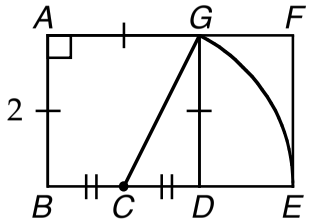
$\frac{\overline{NE}}{\overline{DE}}$ =

4. Measure and find the ratios for the line segment $\overline{GE}$.

$\frac{\text{whole}}{\text{longer}}$ = $\frac{\overline{GE}}{\overline{DG}}$ =

$\frac{\text{longer}}{\text{shorter}}$ = ___

5. Use the lengths of $\overline{CD}$ and $\overline{DG}$ with the Pythagorean theorem to find $\overline{CG}$. Keep your answer in square root form.



6. Fill in the chart.

Lengths	In square root form	In decimal form to 6 decimal places
$\overline{CE}$		
$\overline{DE}$		
$\overline{BE}$		
$\overline{BE}$: $\overline{BD}$		
$\overline{BD}$: $\overline{DE}$		
$\overline{DE}$: $\overline{BD}$		

CONTINUE READING THE LESSON.

Compute your answers to six decimal places.

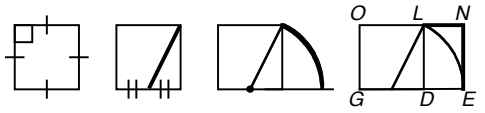
7. What is $\phi + 1$? _____ What is ϕ^2? _____

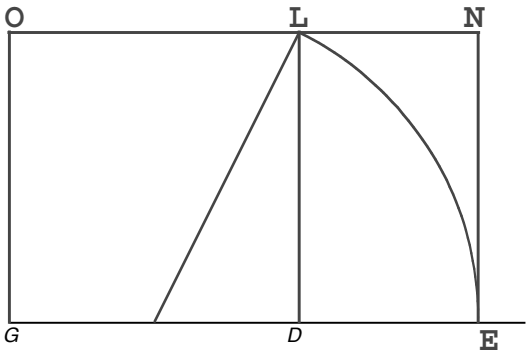
8. What is $\frac{1}{\phi}$? _____ What is $\phi - 1$? _____

9. Does $\phi + 1 = \phi^2$? _____

10. Does $\frac{1}{\phi} = \phi - 1$? _____

1. Construct a golden rectangle on the line below. Follow the instructions provided in the figures. Label the rectangles.





2–3. Measure and find the ratios of the longer side to the shorter side for both rectangles. Round to the nearest tenth.

$\frac{\overline{GE}}{\overline{OG}} = \frac{8.1}{5} = 1.6$

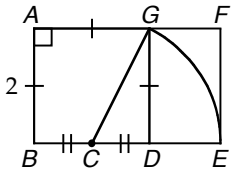
$\frac{\overline{NE}}{\overline{DE}} = \frac{5}{3.1} = 1.6$

4. Measure and find the ratios for the line segment $\overline{GE}$.

$\frac{\text{whole}}{\text{longer}} = \frac{\overline{GE}}{\overline{DG}} = \frac{8.1}{5} = 1.6$

$\frac{\text{longer}}{\text{shorter}} = \frac{\overline{DG}}{\overline{DE}} \quad \frac{5}{3.1} = 1.6$

5. Use the lengths of $\overline{CD}$ and $\overline{DG}$ with the Pythagorean theorem to find $\overline{CG}$. Keep your answer in square root form.



$c^2 = a^2 + b^2$
$c^2 = 1^2 + 2^2$
$c^2 = 5$
$c = \sqrt{5} = \overline{CG}$

6. Fill in the chart.

Lengths	In square root form	In decimal form to 6 decimal places
$\overline{CE}$	$\sqrt{5}$	
$\overline{DE}$	$\sqrt{5} - 1$	
$\overline{BE}$	$\sqrt{5} + 1$	
$\overline{BE} : \overline{BD}$	$\frac{\sqrt{5}+1}{2}$	1.618034
$\overline{BD} : \overline{DE}$	$\frac{2}{\sqrt{5}-1}$	1.618034
$\overline{DE} : \overline{BD}$	$\frac{\sqrt{5}-1}{2}$	0.618034

CONTINUE READING THE LESSON.

Compute your answers to six decimal places.

7. What is $\phi + 1$? 2.618034 What is ϕ^2? 2.618034

8. What is $\frac{1}{\phi}$? 0.618034 What is $\phi - 1$? 0.618034

9. Does $\phi + 1 = \phi^2$? yes

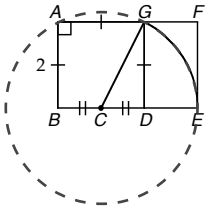
10. Does $\frac{1}{\phi} = \phi - 1$? yes

NOTES: For Problem 1, make sure the student draws a precise square first. If a square is not in place, calculations for 2 through 4 will not be accurate.

If the student is unfamiliar or unsure how to use the mmArc compass, there is a video online at RightStartMath.com/geometry under this lesson number.

When finding the ratios for Problems 2 through 4, watch that the comparisons are longer sides to shorter sides.

Problem 5 can also be calculated as $\overline{CG} = \sqrt{1^2 + 2^2} = \sqrt{5}$, because $c = \sqrt{a^2 + b^2}$, combining steps from the solution shown above. If the student does this, they are doing steps in their heads and is to be commended.



In Problem 6, both $\overline{CG}$ and $\overline{CE}$ are the radius for the arc on the right side of the drawing, therefore, $\overline{CG} = \overline{CE}$. Some students may benefit from continuing the arc to make the circle. This reminds the student that C is the center of the circle, therefore, any lines from the center to the circumference will be the radius, which will all measure the same. See the figure on the right.

To convert the square root form of the answers to decimal form for Problem 6, guide the student to the note in the Extras section of the lesson. When calculating $\overline{BD} : \overline{DE}$, make sure the student realizes 2 is being divided by $(\sqrt{5} - 1)$, not 2 divided by $\sqrt{5}$, which is 0.894, then minus 1 for a total of -0.106. These are two different equations with two very different answers.

DICTIONARY TERMS: golden rectangle, golden ratio, phi, ϕ

Lesson 42: More Golden Goodies

OBJECTIVES:

1. To construct a golden spiral
2. To calculate golden ratios
3. To find the *golden triangle* in a pentagon

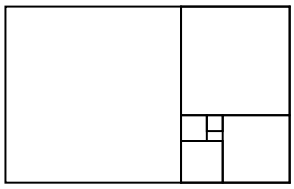
MATERIALS:

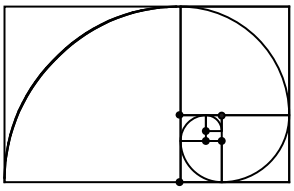
1. Math Dictionary
2. Worksheets 42-1 and 42-2, More Golden Goodies
3. Drawing board, T-square, and 45 triangle
4. Slide N' Measure Compass and 4-in-1 ruler
5. Casio Calculator fx-300MS
6. *Math Card Games* book, A49

ACTIVITIES:

Golden spiral. A golden rectangle can be divided into a square and another golden rectangle. You saw this on Worksheet 41. This process can be continued indefinitely, that is, forever.

The left figure below shows a golden rectangle divided into a square and another golden rectangle. That is divided again into another square and golden rectangle. Can you see them? If you draw an arc that is a quarter of a circle in each square, you will have the golden spiral. See the right figure; the black dots show the circles' centers.



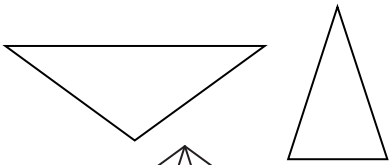


The golden spiral is found in nature, such as in a shell of the chambered nautilus. Astronomers also see it in galaxies. When the spiral is found in nature, it is an approximation of the golden ratio.

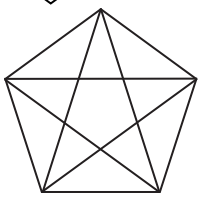
Worksheet 42-1. On the first worksheet you are to draw the golden spiral. The steps are given on the worksheet. Work carefully because in a future lesson, you will draw a similar spiral to compare to this spiral.

Worksheet 42-2, problems 2-5. You are to compare the sides of consecutive squares of the golden spiral. Complete Problems 2 to 5 now, then continue reading.

Golden triangles. A *golden triangle* is a special isosceles triangle where the ratio of the longest side to the shortest side equals ϕ. Two examples are shown in the figures on the right.



Interestingly, a regular pentagon with diagonals has golden triangles. See figure on the right.



Worksheet 42-2, problems 6-9. Now you are to determine whether or not certain triangles are golden. The lengths are given so you can calculate ratios to three decimal places. Complete the worksheet.

Today's game. Play the Long Chain Solitaire game, found in the *Math Card Games* book, A49.

EXTRAS:

This fossil and shell are in a museum in England.

Worksheet 42-1 will be used again in a later lesson.

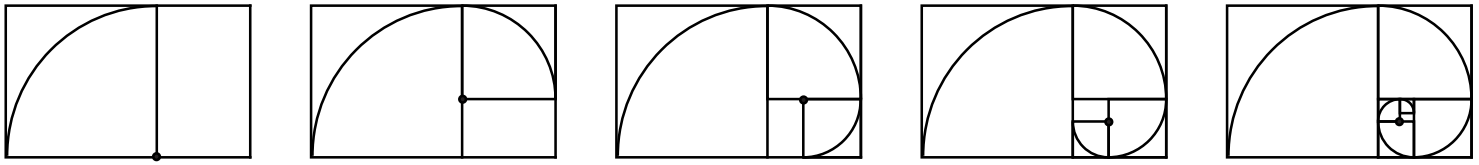
When you complete your worksheet, remember to grade your work on Persistence, Understanding, and Results.

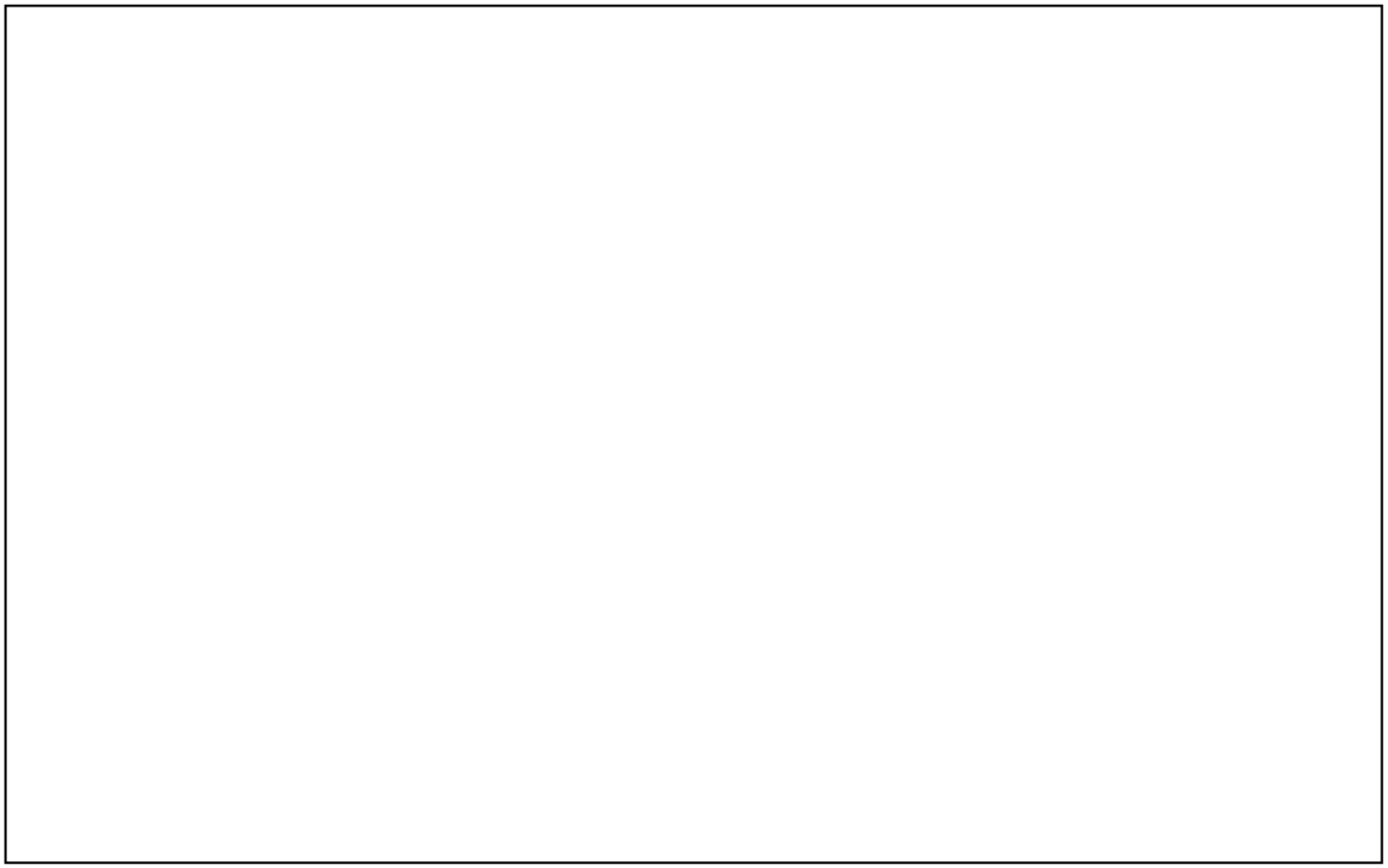
Name: _____

Date: _____

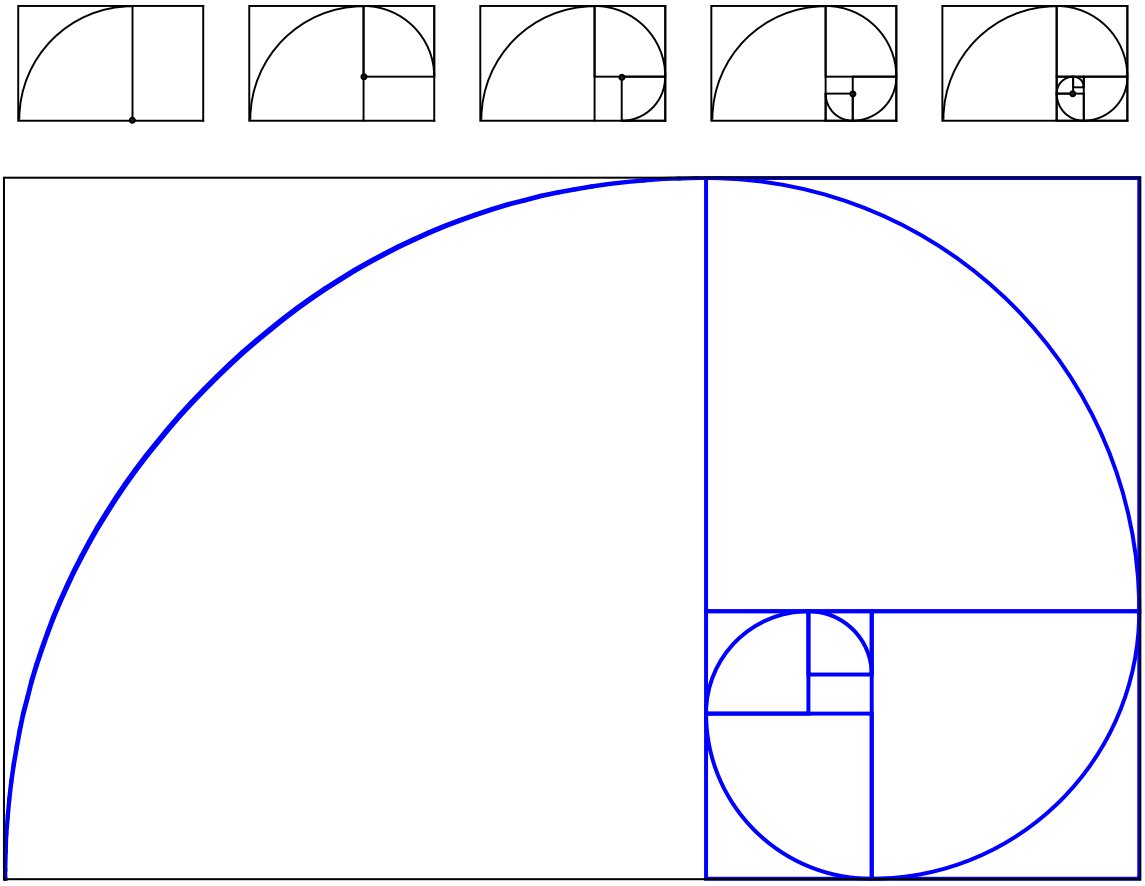
Worksheet 42-1, More Golden Goodies

1. Construct golden rectangles within the golden rectangle by constructing squares. Be as precise as you can as you will need these measurements on the next worksheet. Then draw the spiral. (Because the compass does not allow a radius less than 0.5 inch/1.3 cm, you will need to draw the smallest arc freehand.)





1. Construct golden rectangles within the golden rectangle by constructing squares. Be as precise as you can as you will need these measurements on the next worksheet. Then draw the spiral. (Because the compass does not allow a radius less than 0.5 inch/1.3 cm, you will need to draw the smallest arc freehand.)



NOTES: Accuracy is important with this spiral. Small errors can quickly become cascading errors.

If the student is using the Slide N' Measure Compass with a pencil with a fine point or a mechanical pencil, the hole in the radius slider may be too big and will allow up to 2 mm of variance to occur when drawing the arcs.

One possible solution it to set the slider one millimeter less than the desired measurement, then keep a slight pressure towards the outside of the slider hole when drawing.

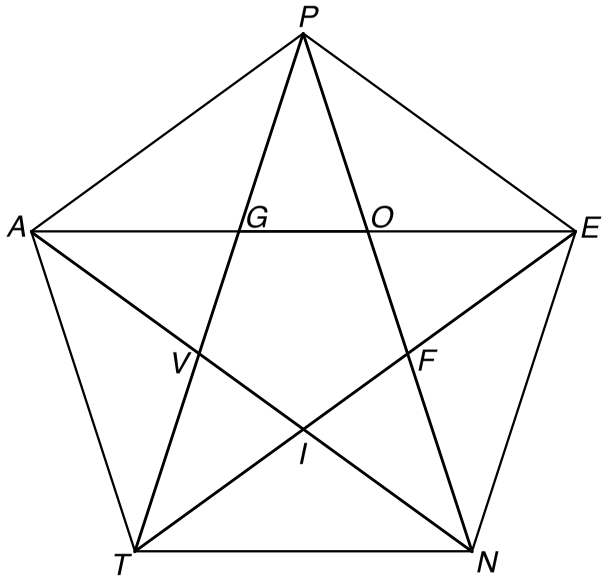
Another possible solution is to use a pencil with a thicker base near the lead, thereby providing support and centering the lead within the slider hole.

Name: _____

Date: _____

Worksheet 42-2, More Golden Goodies

2–5. Measure the sides of the five largest squares you drew on Worksheet 42-1 to the nearest tenth of a centimeter. Find the ratio of the sides of the largest square to the second largest square. Then find the ratio of the sides of the second largest square to the third largest square. Continue for four ratios.



CONTINUE READING THE LESSON.

6–9. Fill in the chart using the regular pentagon above on the right. Find your ratios to 3 decimal places. Use the measurements given on the right.

$\overline{PG} = 6.1803$
$\overline{GO} = 3.8196$
$\overline{AP} = 10.0000$
$\overline{AE} = 16.1803$

Triangle	Longest side/ Shortest side	Ratio	Golden Triangle? (yes or no)
ΔPGO			
ΔPAE			
ΔPAG			
ΔPGE			

10. What is $\angle APE$? _____

11. What is $\angle GPO$? _____

12. What is $\angle APG$? _____

13. Circle the triangles that are similar to ΔEOF.

ΔPAG ΔPAT ΔPTN ΔPGE ΔPGO

14. Circle the triangles that are similar to ΔTIN.

ΔPAG ΔPAT ΔPTN ΔPGE ΔPTF

15. How many golden triangles are in the figure above? _____

16. Would you call *APET* a golden trapezoid? Explain. _____

_____

_____

2–5. Measure the sides of the five largest squares you drew on Worksheet 42-1 to the nearest tenth of a centimeter. Find the ratio of the sides of the largest square to the second largest square. Then find the ratio of the sides of the second largest square to the third largest square. Continue for four ratios.

$\frac{12.0}{7.4} = 1.6$ $\quad$ $\frac{7.4}{4.6} = 1.6$

$\frac{4.6}{2.8} = 1.6$ $\quad$ $\frac{2.8}{1.8} = 1.6$

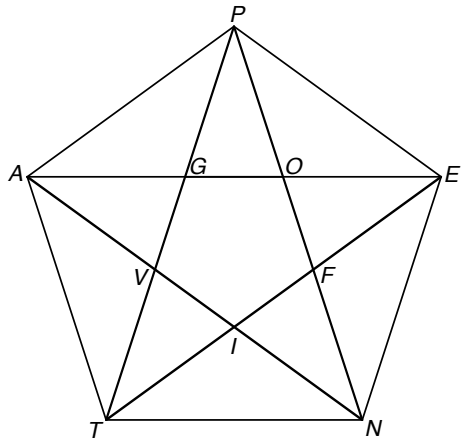
CONTINUE READING THE LESSON.

6–9. Fill in the chart using the regular pentagon above on the right. Find your ratios to 3 decimal places. Use the measurements given on the right.

$\overline{PG} = 6.1803$
$\overline{GO} = 3.8196$
$\overline{AP} = 10.0000$
$\overline{AE} = 16.1803$

Triangle	Longest side/ Shortest side	Ratio	Golden Triangle? (yes or no)
ΔPGO	$\frac{\overline{PG}}{\overline{GO}} = \frac{6.1803}{3.8196}$	1.618	yes
ΔPAE	$\frac{\overline{AE}}{\overline{AP}} = \frac{16.1803}{10.0000}$	1.618	yes
ΔPAG	$\frac{\overline{PA}}{\overline{PG}} = \frac{10.0000}{6.1803}$	1.618	yes
ΔPGE	$\frac{\overline{PE}}{\overline{PG}} = \frac{10.0000}{6.1803}$	1.618	yes

[TRIANGLE SIDES MAY VARY, HOWEVER RATIOS WILL BE THE SAME.]



10. What is $\angle APE$? 108°

11. What is $\angle GPO$? 36°

12. What is $\angle APG$? 36°

13. Circle the triangles that are similar to ΔEOF.

ΔPAG ΔPAT (**ΔPTN**) (**ΔPGE**) (**ΔPGO**)

14. Circle the triangles that are similar to ΔTIN.

(**ΔPAG**) (**ΔPAT**) ΔPTN ΔPGE (**ΔPTF**)

15. How many golden triangles are in the figure above? 35

16. Would you call *APET* a golden trapezoid? Explain. yes
The 3 shorter sides are equal.
The longer:shorter ratio = ϕ.

NOTES: Problems 2 to 5 may have variances with the measurements, thereby resulting in variances in the ratios. The numbers should be close to 1.6.

When working on the chart for Problems 8 to 11, some students find it beneficial to identify the triangles, then consider which side is the longest and which is the shortest. Although the chart identifies the line segment as well as the measurements, it is not necessary for the student to write both. They will need to use the measurements given and notice which line segments are congruent in the figure to find the measurement of the line segments not given.

For Question 12, when finding the measurement of $\angle APE$, remind the student that to find the interior angles of a pentagon, divide it into three triangles (Lesson 6). The total angles of the pentagon is 180 × 3, which is 540°. Therefore, each of the five angles is 540 ÷ 5 or 108°.

Question 13 is simpling dividing the angle of the pentagon by the three triangles; 108 ÷ 3. Some students may wonder how they would know that the three angles are equal. Because this is a regular pentagon, the interior pentagon, *GOFIV*, drawn with symmetrical lines, is also a regular pentagon. The angles on a regular pentagon are 108°. The vertical angle, $\angle PGA$, will also be 108°. Because ΔPGA is an isosceles triangle, the two other angles will be equal; 180 – 108 = 72 and 72 ÷ 2 = 36°. This means $\angle P$ is 108 – 36 – 36, which is 36°!

For Question 18, Bailey Hodson, age 13, answered, "Probably. Maybe. Kinda. Yes, because it looks like it has the right amount of proportion." Although that's not a mathematical answer, Bailey was able to recognize the ratios! Her brother, Seth Hodson said, "Yes, because it is made of golden triangles."

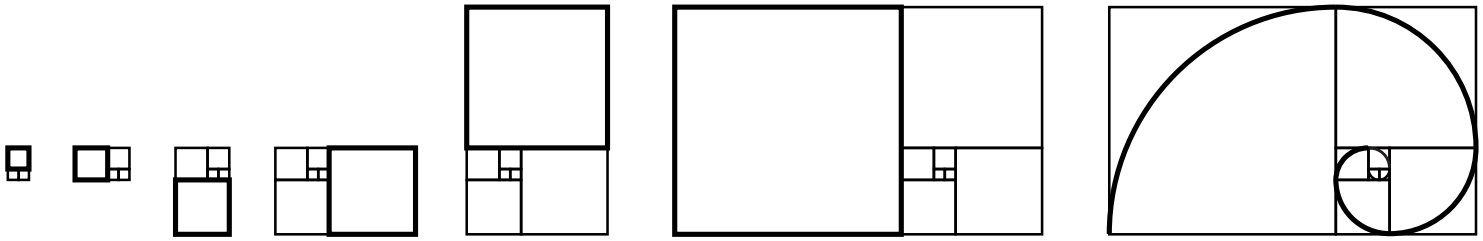
DICTIONARY TERMS: golden triangle

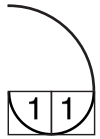
Name: _____

Date: _____

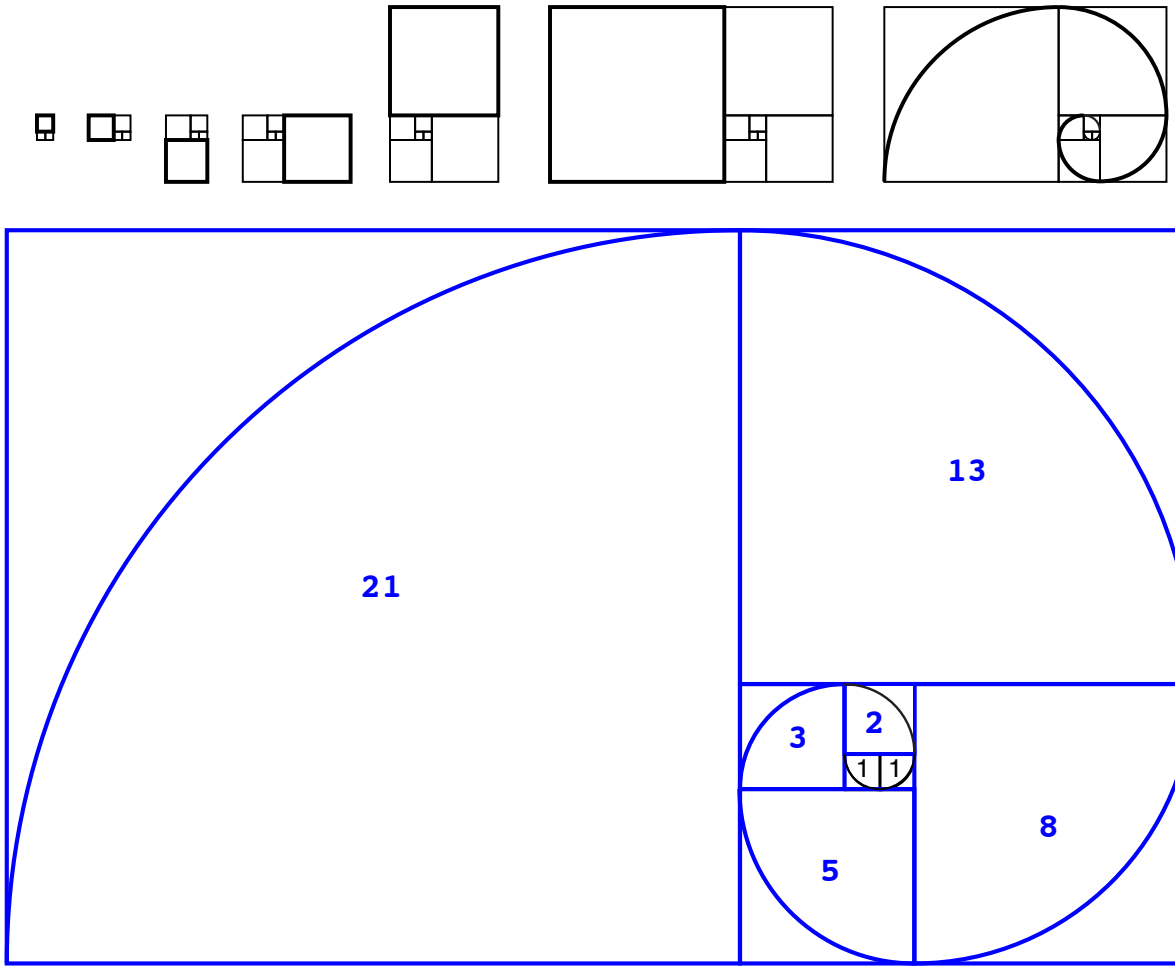
Worksheet 45, Fibonacci Spiral

Construct the Fibonacci spiral as shown in the steps below. Write the Fibonacci number in each square.





Construct the Fibonacci spiral as shown in the steps below. Write the Fibonacci number in each square.



NOTES: Remind the student to use their drawing tools to create the squares (Lesson 5) rather than measuring. For example, the center two squares are given – each represents 1, although they don't measure 1 inch or 1 cm. So the square on top of the two 1s represents a square 2 by 2.

When drawing the spirals, the student could measure the radius, which is the side of a square, or simply place the Slide N' Measure Compass at the corner of the square and find the radius that aligns to make the arc.

DICTIONARY TERMS: none

Name: _____

Date: _____

Write the correct term for the following definitions.

1. A set of quantities in some type of order is called? _____

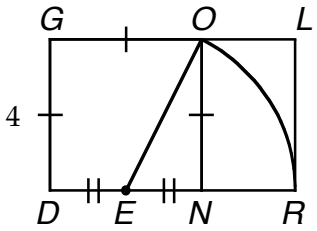
2. The Greek symbol for the golden ratio, 1.618034? _____

3. What is the missing part of the formula for the golden ratio?

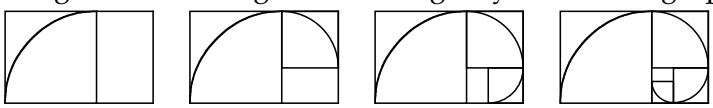
$$\frac{\text{whole}}{\text{longer part}} = \frac{\text{\_\_\_\_\_}}{\text{shorter part}}$$

a. equal part
b. upper part
c. longer part

4. Use the lengths of $\overline{EN}$ and $\overline{NO}$ with the Pythagorean Theorem to find $\overline{EO}$. Keep your answer in square root form. So now, what is the length of $\overline{ER}$?



5. Construct golden rectangles within the golden rectangle by constructing squares and then draw the spiral.



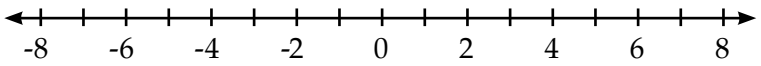
Name: _____

Date: _____

Worksheet 117,
More about Inequalities

1. Write out in words what this inequality means, $4 \leq g \leq 8$ and graph it on the number line.

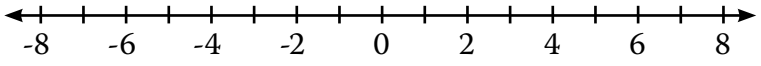
_____



Can g be equal to 8?_____ Can g be equal to 3?_____

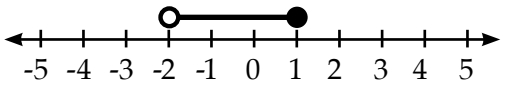
2. Write out in words what this inequality means, $-3 > r \geq -6$ and graph it on the number line.

_____

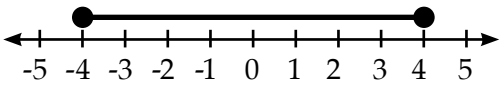


Can r be equal to -3?_____ Can r be equal to -6?_____

3–4. Write inequalities that express what the graphs show.

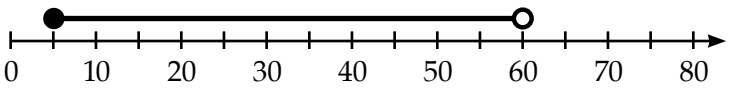


_____



_____

5. The ages, a, for those who pay $10 for Alex's tickets is expressed by inequality, $5 \leq a < 60$. Those under 5 or those 60 or older get free tickets. Write and graph the expression(s) for the ages that get in free.



_____

6. Write an expression for the number of days in a month. _____

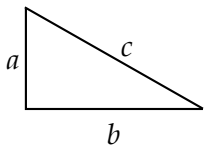
7. On the Celsius scale, at sea level water at 0° can be both frozen or liquid and at 100° can be both liquid or gas. Write an expression for the temperature range that water can be a liquid.

_____

8. On the Fahrenheit scale, water at 32° can be both frozen or liquid and at 212° can be both liquid or gas. Write an expression for the temperature range that water can be a liquid.

_____

9–11. For the right triangle shown, write the correct symbols to make the following expressions true.



$a^2 + b^2$ _____ c^2 $a + b$ _____ c b _____ $c + a$

Lesson 119: Absolute Value

OBJECTIVES:

1. To learn the term *absolute value*
2. To learn the absolute value sign, "| |"
3. To solve equations with absolute values

MATERIALS:

1. Math Dictionary
2. Straightedge
3. Worksheet 119, Absolute Value

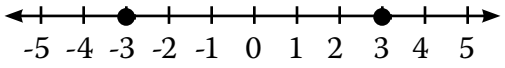
ACTIVITIES:

Introducing absolute value. Absolute value is a fairly simple concept. It is the value of a number or variable without regard to it being positive or negative. The absolute value sign is a pair of vertical lines, one on each side of the number. This sign has been in use since 1841, which is fairly recent for math symbols.

$$|4| = 4$$
$$|-4| = 4$$

The *absolute value* of a number is its distance from 0. This means the absolute value of 0 or any number greater than 0 is unchanged. And the absolute value of any number less than 0 is the number without the negative sign.

Absolute value on a number line. Look at the two points on the number line shown below. Think what their absolute values are before continuing.



$|-3| = 3$ $\qquad$ $|3| = 3$

Worksheet 119, problems 1-5. Do the first five problems, then continue reading.

Solving absolute value equations. Let's start with a very simple example: $|x| = 13$

This means x can be either 13 or -13, making absolute value equations a bit unusual because they have two solutions. Therefore, $x = 13$ **or** $x = -13$.

Now try: $|r - 4| = 1$

Remove the absolute value signs and write the two equations, one with a positive value and a second one with its negative value.

$r - 4 = 1$ $\qquad$ $-(r - 4) = 1$

Solving the positive value equation is straightforward. Add 4 to each side of the equation:

$$r - 4 = 1$$
$$r - 4 + \mathbf{4} = 1 + \mathbf{4}$$
$$r = 5$$

Substitute this value into the original equation. Does it check?

EXTRAS:

Another way to look at this is as follows. For the positive value, you could think: $|x| = 13$
$x = 13$.

For the negative value, you could think: $|x| = 13$
$-x = 13$
which is the same as $x = -13$.

If you are curious how you went from $-x = 13$ to $x = -13$, remember what happens when you multiply both sides of an equation by -1.

ACTIVITIES:

Solving the negative value equation can be done two ways. You will quickly find the way that works best for your thinking, then almost do it intuitively.

The negative equation for $|r - 4| = 1$ is $-(r - 4) = 1$. First, use the distributive property to remove the parentheses:

$-r - -4 = 1$, which is the same as $-r + 4 = 1$

Next subtract 4 from both sides:

$-r + 4 - \mathbf{4} = 1 - \mathbf{4}$, so $-r = -3$

Multiply both sides by -1 and $r = 3$. Again, substitute this second value into the original equation mentally. Does it check?

A second way to solve the negative value equation of $-(r - 4) = 1$ is to first multiply both sides of the equation by -1:

$(r - 4) = -1$

Now add 4 to both sides of the equation:

$r - 4 + \mathbf{4} = -1 + \mathbf{4}$

$r = 3$

This is the same answer that the first method determined!

Worksheet 119. Complete the worksheet.

Today's game. Play Super Corners™. The instructions for this game are below.

EXTRAS:

Refer back to Lesson 107 if you need a reminder on subtracting negative numbers.

Refer back to Lesson 108 if you need a reminder on multiplying negative numbers by negative numbers.

There is a third approach to solve $|r - 4| = 1$. You know that $r - 4$ will be 1 or -1 because the equation tells us that the absolute value of $r - 4$ is 1. This allows the two values to be quickly written as $r - 4 = 1$ and $r - 4 = -1$ producing the answers of 5 and 3.

Super Corners™

Object of the game. To have the highest score. Try to beat your previous scores!

Cards. The 50 Corners cards.

Number of players. One.

Deal. Place the deck of cards face down on the table off to one side. Take five cards and place them face up in front of you. Take another card and place it face up in the center of the table. When a card is played, take another card so that five are always face up to play.

Play. Combine the cards by matching colors and sums equaling multiples of five, 5, 10, 15, or 20. No scoreless matching is permitted. Cards can be played anywhere without regard to the last card played.

Scoring. The scoring is what makes this game different. Cards placed with only one side connected will score that sum.

Cards placed with two sides connected, a corner, will get the total of the two sums multiplied by 2. Cards placed with three sides connected, a bay, will get the cards' sum multiplied by 3. If you are lucky enough to fill a window with all four sides joined, the total sum is multiplied by 4!

The game is over when all the cards are played.

Variation 1. Use the multiples of three.

Variation 2. Use the multiples of four.

Variation 3. Play the game with the Android or Apple app.

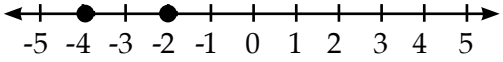
Name: _____ *Worksheet 119, Absolute Value*

Date: _____

1. Evaluate these expressions.

$\lvert -6 \rvert =$	$\lvert 42 \rvert =$	$\lvert -7 \times -11 \rvert =$
$\lvert 6 \rvert =$	$\lvert -18 \rvert =$	$\lvert \frac{1}{3} - \frac{2}{3} \rvert =$
$\lvert 1 - 7 \rvert =$	$\lvert 3 \times -6 \rvert =$	$\lvert \frac{1}{3} - \frac{-2}{3} \rvert =$

2. What is the absolute value of the two points? _____ _____



3. Define absolute value. _____

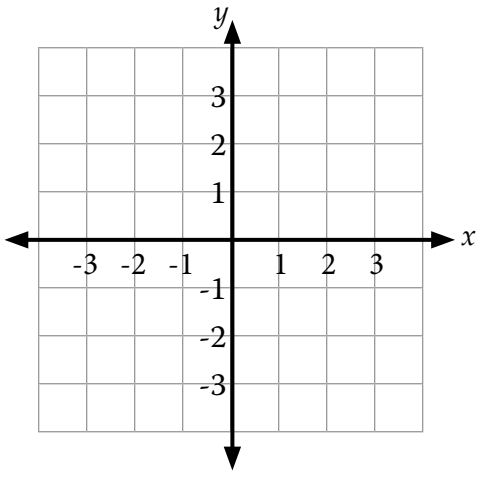
4. Fill in the tables and graph the equations, $y = x$ and $y = |x|$ for x.

$y = x$

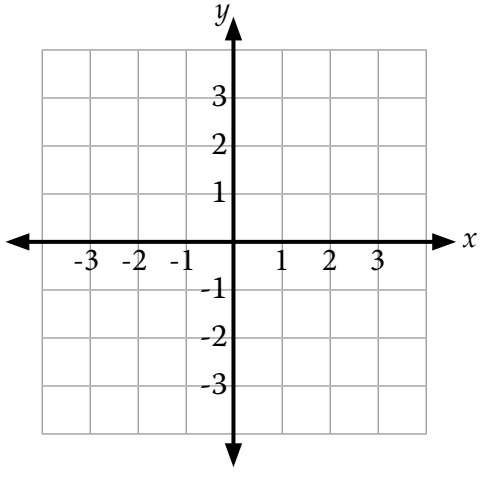
x	y
-3	
-2	
-1	
0	
1	
2	
3	

$y = |x|$

x	y
-3	
-2	
-1	
0	
1	
2	
3	



$y = x$



$y = |x|$

5. Which graph is symmetrical about the y-axis? _____ **CONTINUE READING THE LESSON.**

6–9. Find both solutions for the following equations.

$|x| = 5 - 4$

$|a + 6| = 4$

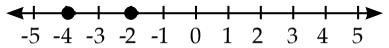
$|2 - s| = 2 + 3 \times \frac{1}{3}$

$|b - 1| = 2(5 - 2^2)$

1. Evaluate these expressions.

$\lvert -6 \rvert =$ 6	$\lvert 42 \rvert =$ 42	$\lvert -7 \times -11 \rvert =$ 77
$\lvert 6 \rvert =$ 6	$\lvert -18 \rvert =$ 18	$\lvert \frac{1}{3} - \frac{2}{3} \rvert =$ $\frac{1}{3}$
$\lvert 1 - 7 \rvert =$ 6	$\lvert 3 \times -6 \rvert =$ 18	$\lvert \frac{1}{3} - \frac{-2}{3} \rvert =$ $\frac{3}{3}$ or 1

2. What is the absolute value of the two points? 4 2



3. Define absolute value. the distance the number is from 0

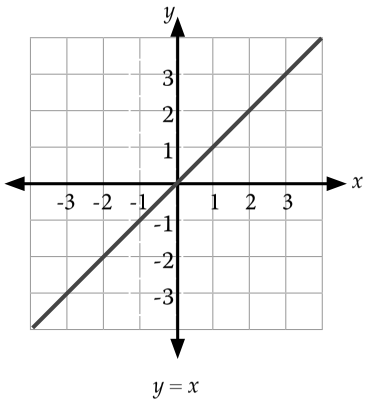
4. Fill in the tables and graph the equations, $y = x$ and $y = |x|$ for x.

$y = x$

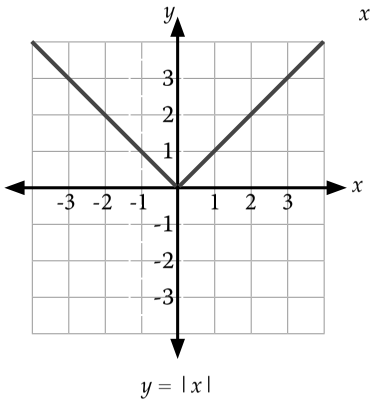
x	y
-3	-3
-2	-2
-1	-1
0	0
1	1
2	2
3	3

$y = |x|$

x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3



$y = x$



$y = |x|$

5. Which graph is symmetrical about the y-axis? y = |x| **CONTINUE READING THE LESSON.**

6–9. Find both solutions for the following equations.

$|x| = 5 - 4$

x = 1 x = -1

$|a + 6| = 4$

a + 6 = 4 a + 6 = -4
a = -2 a = -10

$|2 - s| = 2 + 3 \times \frac{1}{3}$

|2 - s| = 3
2 - s = 3 2 - s = -3
-s = 1 -s = -5
s = -1 s = 5

$|b - 1| = 2(5 - 2^2)$

|b - 1| = 2
b - 1 = 2 b - 1 = -2
b = 3 b = -1

NOTES: For Problems 6-9, some students find it useful to simplify before they split into two equations and then solve for both answers.

Some students miss that two answers are possible; one as a negative number and the second one as a positive number. Encourage the student to "plug in" their two answers into the equation to verify accuracy.

Because there are multiple ways to solve the negative equations, the solutions just show the basic steps. Also, remember that when $-s = 1$, it is the same as $s = -1$. This is a result of multiplying both sides of the equation by -1, which gets the variable as a positive value.

DICTIONARY: absolute value

Name: _____ *Worksheet 123-2, Review and Games 12*

Date: _____

10. Write the relationship symbols to make the following expressions true: <, ≤, =, ≥, or >.

1 cm _____ 1 m 100 mm _____ 1 dm 1 dm _____ 1 cm 28 days _____ 1 month

11–13. Find both solutions for the following equations.

$|t + 4| = 9$

$|x + 2| = 2(9 - 2)^2$

$|n - 5| = 3^2 - 7$

14–17. Fill in the table below. Round your calculations to two decimal points where necessary.

	Data	**Mode**	**Median**	**Arithmetic Mean**	**Geometric Mean**
a.	6, 3, 7, 3, 8				
b.	2, 5, 2, 3, 17, 11				
c.	24, 87, 83				
d.	97, 83, 72, 97				

Given a deck of 50 cards with the numbers 1 to 50, answer the following questions.

18. What is the probability you will choose a number that is a multiple of 5? _____

19. What is the probability you will select a 12? _____

20. What is the probability you will choose numbers that are odd? _____

21. What is the probability you will choose a number that is a multiple of 25? _____

22. What is the probability you will choose a double digit number? _____

23. What is the probability you will choose a zero? _____

10. Write the relationship symbols to make the following expressions true: <, ≤, =, ≥, or >.

1 cm __<__ 1 m 100 mm __=__ 1 dm 1 dm __>__ 1 cm 28 days __≤__ 1 month

11–13. Find both solutions for the following equations.

$|t + 4| = 9$

```
t + 4 = 9          t + 4 = -9
    t = 5              t = -13
```

$|x + 2| = 2(9 - 2)^2$

```
|x + 2| = 98
 x + 2 = 98        x + 2 = -98
     x = 96            x = -100
```

$|n - 5| = 3^2 - 7$

```
|n - 5| = 2
 n - 5 = 2         n - 5 = -2
     n = 7             n = 3
```

14–17. Fill in the table below. Round your calculations to two decimal points where necessary.

	Data	Mode	Median	Arithmetic Mean	Geometric Mean
a.	6, 3, 7, 3, 8	3	6	5.40	4.97
b.	2, 5, 2, 3, 17, 11	2	4	6.67	4.73
c.	24, 87, 83	---	83	64.67	55.75
d.	97, 83, 72, 97	97	90	87.25	86.59

Given a deck of 50 cards with the numbers 1 to 50, answer the following questions.

18. What is the probability you will choose a number that is a multiple of 5? __20%__

19. What is the probability you will select a 12? __2%__

20. What is the probability you will choose numbers that are odd? __50%__

21. What is the probability you will choose a number that is a multiple of 25? __4%__

22. What is the probability you will choose a double digit number? __82%__

23. What is the probability you will choose a zero? __0%__

NOTES: For Question 10, some students may need to write 10 mm = 1 cm and 10 cm = 1 dm. Referencing a ruler may help some students.

For Problems 11 to 13, some students find it useful to simplify before they split into two equations, then solve for both answers. Because there are multiple ways to solve the negative equations, the solutions just show basic steps. Your student may have steps preceding or differing from those shown. If needed, remind the student to "plug in" their two answers into the equation to verify accuracy.

For Problems 18 to 23, many student will start with a fraction, then convert to percentages. A common mistake is to forget that the count is out of 50, not out of 100 which is needed for percentages. The fractions are as follows:

18. $\frac{10}{50}$ or $\frac{20}{100}$ or 20%
19. $\frac{1}{50}$ or $\frac{2}{100}$ or 2%
20. $\frac{25}{50}$ or $\frac{50}{100}$ or 50%
21. $\frac{2}{50}$ or $\frac{4}{100}$ or 4%
22. $\frac{41}{50}$ or $\frac{82}{100}$ or 82%
23. $\frac{0}{50}$ or $\frac{0}{100}$ or 0%

DICTIONARY TERMS: none

Name: _____

Date: _____

Write the correct term for the following definitions.

88. What is the name for the solids having more than one regular polygon type and identical vertices?

_____

89. What is the name for a polyhedron that resembles a star? _____

Write the correct symbols.

90. Greater than or equal to: _____

91. Less than or equal to: _____

92–96. Fill in the table below. Round your calculations to two decimal points where necessary.

	Data	**Mode**	**Median**	**Arithmetic Mean**	**Geometric Mean**
a.	2, 5, 32				
b.	3, 31, 32, 19				
c.	2, 9, 17, 58, 17				
d.	6, 8, 8, 10, 8				
e.	23, 6, 9, 11, 9				

Base your answers to the following questions assuming you have a deck of 10 cards with the numbers 1 to 10.

97. What is the probability you will choose a number that is a multiple of 10? _____

98. What is the probability you will select an 8? _____

99. What is the probability you will choose a number that is even? _____

100. What is the probability you will choose a number that is a multiple of 5? _____

Write the correct term for the following definitions.

88. What is the name for the solids having more than one regular polygon type and identical vertices?

Archimedean solids OR semi-regular polyhedron

89. What is the name for a polyhedron that resembles a star? stella octangula

Write the correct symbol.

90. Greater than or equal to: ≥

91. Less than or equal to: ≤

92–96. Fill in the table below. Round your calculations to two decimal points where necessary.

	Data	Mode	Median	Arithmetic Mean	Geometric Mean
a.	2, 5, 32	--	5	13	6.84
b.	3, 31, 32, 19	--	25	21.25	15.42
c.	2, 9, 17, 58, 17	17	17	20.6	12.47
d.	6, 8, 8, 10, 8	8	8	8	7.90
e.	23, 6, 9, 11, 9	9	9	11.6	10.42

Base your answers to the following questions assuming you have a deck of 10 cards with the numbers 1 to 10.

97. What is the probability you will choose a number that is a multiple of 10? 10%

98. What is the probability you will select an 8? 10%

99. What is the probability you will choose a number that is even? 50%

100. What is the probability you will choose a number that is a multiple of 5? 20%

NOTES: For Problems 97 to 100, many students will start with a fraction, then convert to percentages. The fractions are as follows:

97. $\frac{1}{10}$ or $\frac{10}{100}$ or 10%

98. $\frac{1}{10}$ or $\frac{10}{100}$ or 10%

99. $\frac{5}{10}$ or $\frac{50}{100}$ or 50%

100. $\frac{2}{10}$ or $\frac{20}{100}$ or 20%

DICTIONARY TERMS: none

Name: _____ *Worksheet 125-8, Assessment 3*

Date: _____

Write the correct term for the following definitions.

83. What is the value of a number without regard to it being positive or negative? _____

84. What is the mathematical word for the most used number in a data set? _____

Write the definitions for the following terms.

85. Mean: _____

86. Median: _____

87–90. Fill in the table below. Round your calculations to two decimal points where necessary.

				Estimated		**Calculated**	
	Data	**Mode**	**Median**	**Arithmetic Mean**	**Geometric Mean**	**Arithmetic Mean**	**Geometric Mean**
a.	99, 66, 77, 88						
b.	99, 66, 77, 88, 66						
c.	99, 66, 77, 88, 99						
d.	267, 275, 275						

Base your answers to the following questions assuming you have a deck of 20 cards with the numbers 1 to 20.

91. What is the probability you will choose a number that is a multiple of 10? _____

92. What is the probability you will select a 19? _____

93. What is the probability you will choose numbers that are odd? _____

94. What is the probability you will choose a number that is a multiple of 5? _____

Write the correct term for the following definitions.

83. What is the value of a number without regard to it being positive or negative? absolute value

84. What is the mathematical word for most used number in a data set? mode

Write the definitions for the following terms.

85. Mean: the mathematical word for average

86. Median: the middle number when numbers are put in order

87–90. Fill in the table below. Round your calculations to two decimal points where necessary.

				Estimated		Calculated	
	Data	Mode	Median	Arithmetic Mean	Geometric Mean	Arithmetic Mean	Geometric Mean
a.	99, 66, 77, 88	---	82.5	[ANSWERS WILL VARY]		82.5	81.57
b.	99, 66, 77, 88, 66	66	77	[ANSWERS WILL VARY]		79.2	78.19
c.	99, 66, 77, 88, 99	99	88	[ANSWERS WILL VARY]		85.8	84.79
d.	267, 275, 275	275	275	[ANSWERS WILL VARY]		272.33	272.31

Base your answers to the following questions assuming you have a deck of 20 cards with the numbers 1 to 20.

91. What is the probability you will choose a number that is a multiple of 10? 10%

92. What is the probability you will select a 19? 5%

93. What is the probability you will choose numbers that are odd? 50%

94. What is the probability you will choose a number that is a multiple of 5? 20%

NOTES: For Problems 91 to 94, many student will start with a fraction, then convert to percentages. A common mistake is to forget that the count is out of 20, not out of 100 which is needed for percentages. The fractions are as follows:

91. $\frac{2}{20}$ or $\frac{10}{100}$ or 10%
92. $\frac{1}{20}$ or $\frac{5}{100}$ or 5%
93. $\frac{10}{20}$ or $\frac{50}{100}$ or 50%
94. $\frac{4}{20}$ or $\frac{20}{100}$ or 20%

DICTIONARY TERMS: none